NATURE'S HARMONIC UNITY

A TREATISE ON

ITS RELATION TO PROPORTIONAL FORM

BY

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THE MATHEMATICAL ANALYSIS BY THE EDITOR

PREFACE

MANY years ago I became profoundly interested in the study of the laws governing proportional form, both in Nature and in the arts and sciences, and the further these investigations were pursued the firmer became the conviction that the laws of proportion, as exemplified in Nature, were startlingly uniform, whatever might be the complexity of the application. In the course of these investigations covering a vast number of Nature's forms and phenomena, and of the laws governing proportional form, the crystal was found to be their most perfect interpreter, but in the flower, the shell, and many other forms of varying origin or organism, the same constructive principles were recognized, and these it will be my endeavor in this work to explain. As these principles are the fundamental elements by which Nature creates harmony, correlating the parts of her form-compositions into a perfect whole, they must be of interest to the student of beauty and of use to the professional artist. Many of these laws were unquestionably among the valued and guarded secrets of the Masonic Order and of the ancient guilds, and as time went on were, unfortunately, guarded not wisely but so well that for generations they have been lost to the use not only of the world, but to those very architects and masons who most treasured them as well.

It is upon the application of these same ultimate principles of harmonic proportion, governing every natural formation, organic or inorganic, to music, sculpture, painting, and above all to architecture, that consciously or unconsciously man has laid the lasting foundation of all form-beauty in art; and only by the recognition of these principles can we hope to deduce intelligible and universal laws governing the fundamental Harmony of Nature.

In this investigation I have received much help from Mr. Jay Hambidge, the artist, whose work on the proportions of the Parthenon was so warmly recognized by the Hellenic Society of London, and who was among the first students to disclose the true symmetry of shells, and to call attention to the binary principle of Nature.

To Professor Tyndall I owe my knowledge of polar force, which he describes in his lectures on Light, revealing its influence on angular magnitude, lessons of lasting value to the architect as well as to the artist.

I am also much indebted to the writings of D. R. Hay of Edinburgh, who, like Hambidge, gives attention to the binary principle, or the law of octaves, declaring it to be as important a factor in form-composition as in the science of music. He also called my attention to the value of mathematical curves in formal art as well as applied to the human figure.

Professor Raymond of Princeton University, in his remarkable book on *Proportion and Harmony of Line and Color*, tells us many truths in his absolutely sound treatise on the theories of this subject, disclosing the influence of the laws of repetition, contrast, variety in unity, etc., on art; and Joseph Gwilt, the English architect, has added greatly to our knowledge of the principles of proportion, which he describes at length in his work *On Architecture*.

The combined works of these writers leaves little to be desired for one seeking information on the subject, and it might almost be felt that there is small need for further words to illumine it. My purpose in adding to these, however, is that I may adduce proofs where heretofore only theories have been advanced, since the above authors say but little of Nature's laws in relation to her forms of growth and production, applying their theories to buildings or other works of art, considered most perfect. Therefore when Gwilt, for example, declares that "the square and the cube have been the fundamental forms in past ages for the just development of proportional spaces in art," the words do not impress the mind as of vital value, since no causes or principles of Nature are forthcoming. But the laws of Nature should be the basic element for all theories, and it is chiefly with these laws that I have to deal.

Of late a new light has arisen on the horizon, for the works of Prof. A. H. Church, of Oxford, the distinguished botanist, have disclosed more truth in the relations of plant-growth to proportion than any other of the numerous writers on botany. The principles governing this growth are clearly described in his work on *Phyllotaxis in Relation to Mechanical Laws*, and it is from this source that much of the material in the chapter on botany is derived.

All of the above men have produced theories which not only stir the imagination but also illuminate the pathway leading to a just understanding of Nature.

S. C.

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INTRODUCTION

PROPORTION is a principle in Nature which is a purely mathematical one and to be rightly interpreted by man through the means of geometry; therefore geometry is not only the gateway to science but it is also a noble portal opening wide into the realms of art. Still to a great majority of artists, and to the world at large, the effort to relate science with art is now looked upon with the greatest disfavor and even repugnance, and this accounts in a measure for the overwhelming percentage of immature work which characterizes all branches of art in our times. The architect, the sculptor, the painter, etc., each places too much confidence in what he is pleased to call his "feeling" or "genius," without considering the fact that this feeling or genius would not only become more profound, but capable of a larger expression, were the mind endowed with fuller knowledge of the laws of beauty. Furthermore the eye becomes better trained under the influences of the exact study of geometry, and thus the student is able more readily to recognize and more justly to appreciate the various charms of Nature.

Analysis discloses the fact that the marvellous symmetry which characterizes the crystal has its exact counterpart in the formation of the flower and the shell; while an examination of the masterpieces of art in the past ages clearly shows that the men who produced them were conversant with those geometric principles which lead to unity of expression; and just in proportion as these artists were masters of this knowledge and imbued with this spirit did their resultant buildings, statues, or pictures become ideal in character and worthy the admiration of all the ages.

But in our day art is taught in studios, academies, and schools in a more or less perfunctory manner, no attempt being made to subject the student to any form of mental or scientific training. The Greek artists as well as those of the Middle Ages were men of learning and many acquirements,—their painters were also accomplished architects, sculptors, and goldsmiths, understanding the principles of chemistry as well, and before entering the schools of art were required to have an accurate knowledge of geometry. One of the greatest reasons that we should still have hope in the future for a superior form of art in our country comes from the fact that our people are beginning to manifest an eager interest in the suitable decoration of their public buildings, already developing remarkable results by reason of the wisdom shown in selecting the artists for this work from among the men with a comprehensive and rigorous training, qualities rarely to be met with among those who devote themselves to the production of easel pictures only, for this, if continually persisted in, must eventually lead to a debased art.

The time cannot be far distant, however, when it will be universally recognized that the highest possible plane of work for the sculptor, painter, and architect will be that in which each collaborates with the other, all engaged simultaneously upon and engrossed in one great whole, and all alike endowed with a scientific knowledge of the laws of Nature. Then the easel picture, which should be of secondary importance and not the first as we now make it, will take its proper place in our art history.

The study of the various phenomena in the organic and inorganic world leads, at once, to the just consideration of the principles involved in correlated design, which study depends largely upon measurement, and this, it may here be explained, is the only way mathematics can have to do with art. Because of the prejudice arising from a misconception of terms, when mathematics has been mentioned in connection with æsthetics, the study of form-phenomena has been as a closed book to the artist, he considering that an equation must be found for every part of a design-composition: thus confounding "measurement" with "calculation," terms of very different meaning. The justice and accuracy of the rule adopting a measurement or adapting it to a certain purpose may be demonstrated by calculation; but, the reasoning once scientifically laid bare, the artist or designer may have the advantages of the measurement constantly before him, relegating the abstruse calculation from which it arose to the obscure corner from which it emerged to serve its purpose. It is upon his measurement, not upon calculation, that the artist or designer is largely dependent for a just arrangement of details, as well as to a general conception of the mass. The harmonic relations in Nature and Beauty are fixed beyond all change by him. He has but to study them. Thus it is that a geometrical construction in a design will appeal both to the eye and the mind by its reasonableness.

CHAPTER I

ARGUMENT

"Order is Heaven's First Law."

WHAT does the term "Order" imply in this connection? It would seem to imply the just correlation of each of the parts of an object with the whole. This means that all of the details of a form in Nature are accurately united in perfect harmony, but the scheme of this unity can only be determined with precision by the use of mathematical measurements, angles, points, lines, and surfaces, produced by geometric principles. Unity is the highest element of beauty, and there can be no question but that the laws of growth in Nature are the fundamental ones which govern it. If it be one object of man's use of geometry to investigate the stars in space, the same science may be also similarly employed in an analysis of the laws of plant growth, polar force, or of any of the many other evidences of Nature's handiwork, as the underlying principles are the same in each. We must not consider that the circle, the triangle, and the square are simply forms, but elements representing the divine grammar of Nature. These geometric principles continue throughout the universe in orderly and exact methods, not only to secure beauty of proportion but the highest use as well.

> "The heavens themselves, the planets and this centre,
> Observe degree, priority, and place,
> Insisture, course, proportion, form,
> Office, and custom, in all line of order."
>
> TROILUS AND CRESSIDA.

This influence has always been felt, not only by the most civilized races, but by untutored savages as well; man's history has been written in it for thousands of years, from the axehead of the Stone Age up to the mighty Pyramids of Gizeh, which have stood in their geometric form longer than almost any other work of art, dominating by their simple majesty the minds of passing generations.[1]

[1] It was about the time of the building of the Pyramids of Gizeh that the Pentagram of the Magi became the vogue, and not only did architects study it, but it became a mystic emblem among the educated classes which was applied to all manner of subjects.

The people who created them were among the first to consider philosophically the attributes of the circle, the triangle, and the square, recognizing that they contained the elements for the true measurement of proportional spaces. This knowledge, imparted to the Greeks by the great mind of Pythagoras, after his sojourn in Egypt, enabled their artists and architects to create that perfect proportion and ideal refinement of form which were the distinguishing traits of the art of that period and still remain unsurpassed. The Gothic architect and artist re-discovered many of the Greek secrets, causing beauty once more to bloom in statue, picture, and design, while cathedrals, the perfections of which are still the wonder and admiration of all classes of men, arose to glorify the land. Under the influence of this exact knowledge even the simplest village church of that period charms the world to-day by the perfect balance of its parts. Freemasonry was alive and active then, but its mysteries were known only to the brotherhood. So inviolate were its secrets that we can now learn little of the principles of proportion even from dissertations on the subject in Encyclopædias and other writings, beyond a few set phrases. Even in that remarkable controversy between Ruskin and the architects of his day, not one word was given on either side beyond similar trite sayings.

Moreover, the mistake which is made by many people, and it is a vital one, of questioning the value of the geometric theory of proportion, is this: that they consider geometry to be an invention of man involving his formally constructed equations only. I need hardly say, however, that the principles are not man's invention, although the outward form of Algebra, which is one of its visible interpreters, may be. The truth is that the system is simply the logical expression of one of the methods man has borrowed from Nature by means of which he can more easily investigate scientific questions. It is as much a part of her as the very air we breathe. Its use in an analysis of proportion is like the application of a solvent which must be suited to the object to be resolved, otherwise labor is in vain. In order to prove the statement that geometric correlations and the proportions of objects in Nature are one and the same thing, all that one need do, even though examples are innumerable, is to take a drop from a pool of fresh or salt water for examination under the microscope, when various beautiful creations will appear, revealing in their individual forms all five of the regular polyhedra, exactly as though drawn with mathematical instruments.

It is well recognized that light, sound, and heat are geometric in their action and continue on similar lines of "force" to be amenable to accurate calculation by measurement or numbers, the elemental source of all harmony. By what right then have architects or artists in our time considered that they have any peculiar privilege of their own to seek to construct beauty without an accurate understanding of the same system by which the loveliness of all forms of beauty in Nature comes into existence?[1]

[1] Albrecht Durer (1471–1528) was the first artist of whom we have any record, to write on the theory of proportion, and among other things applying to the subject said: "I have heard how the seven sages of Greece taught

The cant phrase of the day, that "great art can only result from a passionate spontaneity of feeling, and this feeling must be smothered or paralyzed by the employment of scientific methods," comes from a strange misapprehension of the truth. But when to this false statement are added the words, "Hellenic art, in particular, was the outcome of genius under the influence of impulse only," one cannot help thinking that those repeating this idea must be under the influence of blind prejudice, or suffer from a want of knowledge of form-composition or of the facts of history. If Ictinus and Phidias could arise from their graves they would be the first to repudiate and resent such language as applied to their works, for the Parthenon, even with its statues and ornaments in their ruins, is the most perfect form of art that the hand of man has produced, and an analysis of its portico alone will discover the fact that the relation of the details to the mass is one of most perfect unity, owing to the intelligent application of the law contained in the "Progression of the Square," or by the same sure method employed by Nature in her plan of developing the wonderful beauty of the crystal.

Under the influence of exact scientific study the artist will find that his impulses will become more vitally alive, while his imagination as well as intuitive feeling will be greatly strengthened and his achievement become vastly superior and of lasting credit. The sculptor or painter who from want of education or lack of thought condemns mathematics should bear this fact in mind, that the bolder, the swifter, the more impulsive the stroke of the modelling tool or the brush in the hand of a master, the more perfect, the more beautiful, and the more like Nature is the resultant curve, for the arm and hand have become, through training, a sort of mathematical instrument with an exact though complicated radius from which he cannot escape.

From a similar want of thought Beauty is now considered an element impossible to describe in any adequate and conclusive way, while many philosophers and æstheticians agree with Johnson and his intimate friend Reynolds, the artist, who were among the first to declare, "that Beauty exists only in the mind, and just as this mind is prejudiced by its education, will an object be beautiful to one and not so to another. Who shall judge?" A frequently used definition of the word "Beauty" is to the effect "that whatever pleases the eye or the mind is beautiful," but this idea alone is of a kind that might be discussed until doomsday without arriving at any conclusion. The word "Beauty" has come, however, through centuries of use by educated people, to mean something far more than is usually given by lexicographers; very

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that *measure* is in all things (physical and moral) the best." "Those arts and methods which most truly approximate to exact measurement are the noblest."

Edgar Allan Poe, a man to whom the term "genius" seems eminently fitted, said, in his work on the *Rationale of Verse*, that "one tenth of the art of poetry is ethical, but nine tenths appertains to mathematics."

This fact should also be remembered, that Music is the most mathematical of all the arts, yet there is no other one of these that leads the mind so persistently to the highest realms of the imagination, and to that ideal land where passionate feeling finds its most perfect life and expression.

many feel that it is the highest manifestation of the Creator, revealed in mountain, cloud, and ocean, with the countless living things that they contain. But it is only through an accurate analysis of these various forms that a clear and distinct idea may be obtained, where no sophistry in argument can change the result. In this analysis we learn conclusively that the essence of "Beauty" is unity and where unity exists it can be clearly proven, remaining no longer a question of what this man thinks, or that, whose prejudices have blinded his faculties of observation.

It is true that one person may prefer the beauty of man, another one that of woman; I look with especial favor on the rose, while my neighbor takes more delight in the lily; both are right, as all are beautiful. That is, they are objects expressing divine unity of construction, or the perfect correlation of parts in form-compositions.[1] This book is not devoted to the philosophy of beauty, these arguments are only advanced to suggest the idea that future investigators may find, through scientific methods applied to Beauty, a clearer means of interpreting the term than now exists, while these methods will ultimately expose the fallacy, now so generally looked upon as truth, that Science and Beauty are antagonistic. The more profoundly the mind considers the question of Science and Beauty the more deeply must man be impressed by Nature's methods, and under the influence of the resulting knowledge will the heart of a passionate artist be more deeply kindled with a fire which must result in work more perfect and more enduring; for the great law of numerical harmonic ratio remains unalterable, and a proper application of it to art will never fail to be productive of good effect, as its operation in Nature is universal, certain, and continuous. She appears never to tire in following out mechanical principles in her works, and her methods should be properly respected by man when he seeks to construct forms of beauty, while his theories of art should be duly founded on her laws, so exact and infallible.

In showing how the rules and principles of harmonic proportions apply to the arts and sciences, we must not look for blind adherence in every instance to the degree, minute, and second of every angle, and no vernier scale nor micrometer is needed to prove or disprove the propositions here set forth. The acid test of mathematical precision and calculation has been applied to all of the fundamental principles which are claimed, and the result proves itself at every step to be well within the limits of Nature's constant variations, as has been indicated from time to time in the Appendix, and if, in the "multiplicity of instances," cases of slight deviation be pointed out, I may venture the statement that the deviation will prove negligible and serve only to sustain the spirit of the rule.

[1] In a remarkable work, *On a Theory of Pure Design*, lately published by Prof. Denman Ross, of Harvard University, the term Beauty is thus described: "We look for it in instances of Order, in instances of Harmony, Balance, and Rhythm. We shall find it in what may be called supreme instances. It is a supreme instance of Order intuitively felt, instinctively appreciated." As Order, including Harmony, Balance, and Rhythm, can be scientifi-

As Nature loves variety, so she produces unexpected results by combining various forces, and while seemingly she deviates continually from the letter of her harmonic laws, she never does from their spirit. We must not look for an uninterrupted identity of mathematically derived rules with natural phenomena. The forces at work are so varied in their application that no one of them is uninfluenced by others. Botanically, torsions will be found occasionally to displace symmetry in a flower, seismic disturbances will interfere with geologic formations, or the solar influence may destroy the ellipse of an approaching comet, but in no case, actual or imaginary, does Nature show any tendency to anarchy. She abides by her rules, though through the veil of composite effects they are sometimes hard to trace.

What Nature permits, man may surely do, and it is doubtful if a clearer example of the necessity of applying to art Nature's compensatory rule of "give and take" could be offered, than the following.

[1] It may safely be said, without fear of contradiction, that of all the Arts, Music is far and away the most mathematically exact, and in this I do not refer to the question of musical "tempo," which is comparatively simple, but to the rules of vibration and tonal production which govern the relationships of the members of the diatonic scale, and the harmony of composition.

Any studious musician will understand that the natural and perfectly constructed scale, true to the ear and to its mathematical proportions of vibration, contains perfect octaves, perfect dominants, mediants, and other intervals, but he will also understand that this natural and perfect scale is *susceptible of no use whatever* in any key except that of the tonic for which it was constructed. Its tone intervals, as established by the ear and by science, are not all equal, nor are its half tones equal. For example, the ratio of vibration between the tonic and its second (a whole tone) is not the same as between this second and the mediant (also called a whole tone), for if the perfect scale for a single key be examined, the interval from the tonic to the second will prove to be in proportion to the interval between the second and the dominant in the ratio of 51:46 or thereabouts, and *not* according to precise equality. Naturally, then, if the perfect scale based, for example, on "C" as a tonic, be used for the rendition of anything written in the key of "D," it will be found that the interval from this new tonic to its second cannot of course be the perfect first step required, since the interval to be utilized has already been restricted to the vibratory difference indicated for the second step in the "C" scale, to wit, we are obliged to use the smaller of the two so-called whole tones, whereas in this new key of "D" we need the larger whole tone for the first interval. The farther the process is carried, the worse will be found the result, except that every octave is a repetition of every other. The ideal and perfect scale for the tonic "C" thus becomes utterly useless for a modulation into any musically remote key.

cally defined, we therefore recognize that the term Beauty is clearly and accurately set forth by this Master of Arts.

[1] The remaining paragraphs are taken from an article by the Editor.

To adapt any keyed instrument to general use, therefore, Nature's principle of compromise has been utilized in setting what is technically called an equalized "temperament," the greater, intervals, representing the greater ratio of difference of vibration, yielding to their lesser neighbors sufficient of their surplus so that all of the so-called whole tones represent equal, or nearly equal ratios of difference, the half tones being treated in the same way, thus leaving no key in its primary perfection at the expense of every other, but permitting all to be alike possible and pleasing. Here, if anywhere, we surely have an example of a universally adopted, measurable variation from mathematical perfection, under a system of compensation which produces a result both artistic and at the same time eminently practical.

From all of these facts it seems clear that, while Beauty is of many kinds and has many exemplars, her form is always controlled by Mother Nature, and if, by studious observation, man can re-learn a few of the principles employed by Nature in the creation of those forms which are universally acknowledged as Beauty, perhaps the determination of what is beautiful in art, music, architecture, and painting may be made simpler, more certain, and less subject to the whim and fancy of temperamental minds and the fluctuating personal equation.

CHAPTER II

CORRELATIONS

Correlations of Numbers[1]

THE examination of the relations of numbers to each other has been a favorite pursuit of the logicians and mathematical philosophers from time immemorial, and it would seem to be more or less apt that, before endeavoring to analyze the harmonies of substance we should first briefly consider the relations and harmonies of the machinery by which these concrete entities are measured. Let us understand our "yardstick" before we begin to cut our cloth, proceeding in an orderly manner, first to study a few of the fundamentals of numbers in the abstract, then to ponder the conspicuous harmonics of geometrical proportions as adopted by our Mother Nature, turning then from the generic to the specific, to examine her wonders and to see how constantly she applies these proportions; and finally we may learn in what manner these same harmonics of geometry—the veritable "Harmonics of Nature"—have been playing their part in architecture and the arts.

It is not, therefore, the intention, under this head, to treat of concrete numbers as adopted by Nature or man, or as governing factors in any science or art save in the pure science of numbers themselves. Other features will be fully developed in future chapters where the several divisions will be separately taken up, a branch at a time. For the present, let us see what are a few of the harmonic relations of mere quantity to quantity, or of numbers *as such* to each other, rather than to the things numbered.

We shall find, then, that numbers, taken abstractly, bear certain obvious and sometimes harmonic relations to each other regardless of concrete objects. Let us first examine certain individual numbers for their traits and characteristics, for these are as different as the characteristics of our human kind. Since the most ancient times the number representing unity or *one* has stood apart from all others. The ancients, from the time of their first and crudest observations, have set the number *one* in a class by itself for reasons both numerous and conspicuous. Acting upon itself, the number *one* is self-propagative, and in a sense it is its own cause and its own effect, for if it be multiplied by

[1] By the editor.

itself, it produces itself ($1 \times 1 = 1$); when divided by itself, again the result is itself ($1 \div 1 = 1$); subjected to the processes of involution, it remains unchanged, since the square, the cube, or the n^{th} power of one is one; and under the influence of evolution it is still unaffected, for the square root, the cube root, or the n^{th} root of one is still *one* perpetually *one*. Its reciprocal is again itself ($\frac{1}{1} = 1$), and while it is an exact divisor of all other integers, so also are all other integers its exact multiple. It is thus invariable in its effect upon itself except through the processes of addition and subtraction, and even under these, its effect is like no other number, for if it be subtracted from itself, it leaves no quantity, but is transformed from a finite term at once to infinity, thus immediately carrying it outside the realms of all measurable calculation, and leaving addition as the only practical process of its self-variation. Hence *one* is unique among all numbers, as indeed the etymology of the very word "unique" proves.

Furthermore, the most ancient observers realized that, as a measure, the number *one* was capable of indicating *position* only, and neither direction, surface, nor content.

The number *two* has individualities less marked, it is true, than the number *one*, but two features alone insure its recognition, and have set it apart from all antiquity. First, then, it is the result of the earliest mathematical process, that of adding one to one (as we have seen, the only manner in which the unit is self-changeable), and secondly, taken as a measure, a term of two members is competent to show not only position, but direction or distance as well, this quantity having both a fixed start and a fixed termination. It is also the means of dividing all quantity into the two great classes of "even" and "odd."

The number *three*, being the lowest number to have a fixed start, a termination, and also a middle point, is vastly important as representing the smallest or lowest number of members capable of enclosing or outlining any portion of a plane surface. It is also the first and lowest number with which we meet having the capacity of a compound, being the result of the double process of adding $1 + 1 + 1$ or $2 + 1$, the latter being already, as we have seen, a combination.

The number *four* is the first and lowest number met with which is the product of any two integers not immediately influenced by the peculiar properties of the number *one* (2×2). *Four* is also the first and lowest number divisible by any other than itself and one; it is, thus, the lowest number not "prime." It is the first and lowest integral result of involution, being the smallest square (excluding the number *one*, which as we have seen is its own square). The number *four* is, as well, the lowest integer susceptible of perfect evolution (the square root of four equals two). The number *four*, moreover, is fundamentally important, inasmuch as it represents the smallest number of members capable of enclosing space, for even the triangular pyramid or regular tetrahedron must have one base and three sides.

These four were the principle elements in the Pythagorean system of fundamentals, and were designated as the "monad," the "duad," proceeding from the union of monad and monad, the "triad," proceeding from the union of monad and duad, and the "tetrad," proceeding from the union of the duad with the duad. These four are the only terms presenting individual characteristics requiring our examination, and we may therefore proceed to consider numbers in groups or series.

For the purposes of this examination we shall confine ourselves mainly to the various numerical progressions, for it is with these that we have here to do. The simplest of all of these forms is a mere numerical order, 1, 2, 3, 4, 5, 6, 7, 8, 9, etc. This is so axiomatically a progression (by the addition of one each time) that it is usually omitted from classification. It, however, conforms to every requirement of an arithmetical progression and is the most fundamental of all. For the moment, let us pass to the more customary examples of this class such as 2, 5, 8, 11, 14, etc., where the numbers increase by equal intervals or additions (here of 3 each time). It will be observed that, given several factors or members of the series, the mind, in simple cases, supplies the rest intuitively, as would the eye if the series were one of spaces, or the ear, if recurrent sounds were involved. The same would be true were the intervals those of multiplication instead of addition, where the series, for example, would appear as 2, 4, 8, 16, 32, etc., thus coming under the definition of a geometrical progression instead of an arithmetical one. It is perhaps unnecessary to state that if these series were complex instead of the simple examples presented, they would nevertheless be quite as submissive to the rules for their solution, and the value of any unknown term could be immediately predicted, having the rate and other elements given, since these progressions are all of them regular and harmonic.

Besides the two conspicuous forms already referred to, there are many others coming generally under the head of "recurring series." In the ones we have so far examined, every interval is proportional to *every* other, but we find the recurring series presenting certain "free lance" characteristics, since in them *each* interval need not be proportional nor equal to *each* other, provided that after the appearance of one or more intervening members, the same intervals recur in the same order, as for example, 2, 6, 8, 11, 13, 17, 19, 22, etc., in which the respective intervals are seen to be 4, 2, 3, 2, and then repeating, 4, 2, 3, 2, again.

We now arrive at the great point to be attained in thus examining numbers, for of all of the various form of series, the most interesting by far is the one which we shall here take up, and which will well repay study. This series arises out of the natural inclination of the mind to combine any two things under consideration, using the result as a new step or member, to be in turn combined again. This is a well recognized tendency and such a process naturally commences where all numeration begins, with the unit, and grows thus: $1+1=2$; $2+1=3$; $3+2=5$; $5+3=8$; $8+5=13$; $13+8=21$; $21+13=34$, etc., and thus continuing to combine each pair for the production of a new member. To those versed in such matters I need hardly say that this series has been much treated of and has been utilized under more than one name. It will be recognized by many as the well known "Fibonacci Series" and bears other appellations as well; but named or nameless, either in the form of whole numbers as just explained or in the more perfect ratio of a pure extreme and mean proportion, we shall find it flung broadcast throughout all Nature, as will be clearly shown in the subsequent chapters.[1]

This recognized series will be seen upon examination to present the nearest integral equivalent to what in the exact science of mathematics is known as "Extreme and Mean Proportion," which may be defined as *the division of any quantity into two such parts or portions as that the measure of the lesser part shall bear the same relation to the measure of the greater part as the measure of the greater part bears in turn to the measure of the whole quantity.* In pure mathematics this interesting proportion produces endless decimals, but Nature knows no fractions and is bound by no decimal divisions, for she produces her harmonies with a free hand and with inimitable perfection, being able to measure her distances and proportions with the extremest mathematical nicety yet without being subjected to the need of the cumbersome calculation which man would be obliged to use in her place. Nature's protractor is always right, and in her use of infinite subdivisions the smallest humanly conceivable fraction would seem to her a whole number. For ready service, therefore, this continuing series of 5, 8, 13, 21, 34, 55, etc., will be found to be of immense use where absolute exactness is not requisite, since by the employment of these whole numbers the enormous effort required to handle the large fractions otherwise involved in precise extreme and mean proportion is escaped. If we bear in mind, further, that the ancient systems of mathematical numeration were incapable of handling fractions with our modern facility, we shall understand more clearly why the Egyptians and Greeks did not treat fractions as "numbers" and why they so habitually substituted an integral approximate in place of our decimal precision.[1]

In the subsequent chapters we shall find that Nature uses this as one of her most indispensable measuring rods, absolutely reliable, yet never without variety, producing perfect stability of purpose without the slightest risk of monotony.

Before leaving this abstract subject of Nature's measurements it would be well to bear in mind one other principle invariably employed by her. We should remember that, while Nature selects her harmonies to suit herself, one to govern phyllotaxis, another to control all formations subject to polar force only, and so on throughout the list, yet, *once decided on, she never forgets, never changes her mind.* All changes of natural form or substance are the outgrowth of *combinations* of causes which produce a result different from the uninterrupted effect of either cause working alone, and it is a recognized fact that in whatever proportional group a form or process or chemical reaction belonged at

[1] For those interested in understanding the pure mathematics of extreme and mean proportion, the explanation contained in the Appendix note under that head will not be amiss.

the beginning of time, we may be certain that the identical conditions will invariably produce the identical result till the fulfilment of all things.

Correlations of Form

We have seen that the great or primary cause of harmonic unity is a simple one, consisting in the reduplication of the original unit producing the ratio of $1+1$, which is synonymous with the ever present law of repetition, and at certain stages represents a system of "octaves" closely corresponding to the theory of music. It is the basic element of all growth in Nature, guiding her forms to their ultimate use and beauty. Molecular structure, for example, is made up of repeating particles or octants, which would be round if it were not for atmospheric pressure, but under the influence of this principle molecules are packed in the only possible way without interstices or waste room, as shown for example in Plate No. 1, diagrams A and B. If masses of these circles are drawn on a small scale and regarded with the eye partly closed they will appear to be actual hexagons. Diagram B represents the small circles in A pressed into their ultimate form, and in every seven of these circles seven hexagons result. This primal arrangement is under the inviolable influence of "polar force" causing atom to lay itself to atom in a definite way but liable of course to accident; the law is superior to accident, however, for as we shall see *angular magnitude is a principle ultimately enforced.* Thus Nature from the very beginning reveals herself as an architect, building in accordance with the established law of gravity, and angular magnitude becomes at once the fundamental element of proportional form. On this subject there can be no question of doubt. Whenever we examine any form produced by Nature in which she has employed the agency of polar force, we may count on finding the design to be one in which the conspicuous angles will be those of the most prominent aliquot parts of the circle; that is, the quadrant, of 90°, the eighth, of 45°, the sixth,

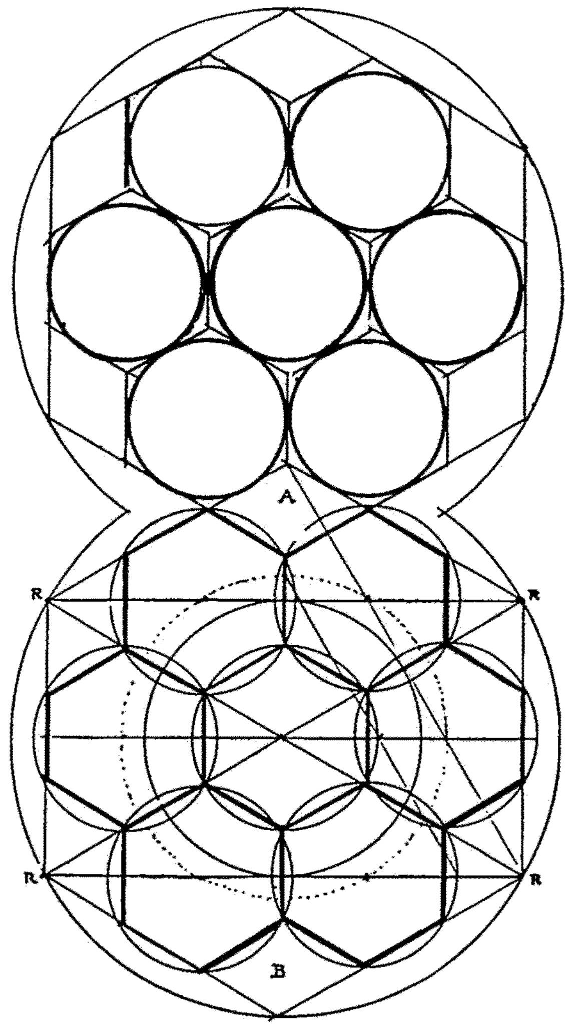


Plate 1—Molecular structure

of 60°, and its complement, of 30°. These constitute the chief angles in the "geometry of space" and are clearly represented by the cube, the square, and the hexagon with which the pentagon is harmonic and by the regular progressions of these figures. From them the true principles of architectonic design may be deduced, to be followed by perfect results according to the genius of the designer by whom they are employed. The angles of 30°/60° produce all crystals numbered by six, such as snow crystals, while that of 45° decides the proportional spaces of all permanent chemical crystals, rarely failing to do this with an exactitude and perfection of proportional spaces unsurpassed.

The influence of these angles may be distinctly traced not only in the Greek temples, both before and after Ictinus, but also in the Gothic buildings of the Middle Ages; and as competent judges unite in declaring the Parthenon to be the most perfect work by the hand of man, I have employed it as a text or test in many places in these pages, for purposes of comparison with spaces resulting from various geometric correlations applied to works of art. So far as the works of Nature are concerned it would be difficult to say where the angles of polar force cease to exist as the great factors in contributing to the beauty of form, line, and curve, from the waves of the sea to the clouds, the trees, and the flowers. Even the magnificent forms of mountains owe their modelling and line to this law. The catenary curve is the most perfect of all curves and can be traced from the very beginning of things. Its manifestations can be better described mathematically than in any other way and its principle may be equally well applied to other geometric forms, outside of the angles of polar force, for the harmonic relation between them is most perfect.

On the Polyhedra

The Greeks, long before the Christian era, discovered that there could be but five regular solid bodies, or polyhedra, three of these formed by the use of the equilateral triangle, one based upon the square of 90°, and one upon the regular pentagon, which forms we will now analyze as set forth by Euclid.

The simplest regular polygon is the equilateral triangle, and since each apex of an equilateral triangle is an angle of 60°, three such triangles can be combined to form a polyhedral angle. It is seen, then, that a regular polyhedron can be formed, bounded by equilateral triangles and having three at each vertex. This has four faces and is called the tetrahedron. (See fig. A, Plate 2.) Since four angles of 60° are less than four right angles, four equilateral triangles can be combined to form a polyhedral angle and it is seen, then that a regular polyhedron can be formed, bounded by equilateral triangles and having four at each vertex. There is such a regular polyhedron. It has eight faces and is called a regular octahedron. (Plate 2, fig. B.) Since five angles of 60° are less than four right angles, five equilateral triangles can be combined to form a polyhedral angle, and it is seen then that a regular polyhedron can be formed, bounded by equilateral triangles and having five at each vertex. There is such a regular polyhedron. It has twenty faces and is called the regular icosahedron. (Plate 2, fig. C.) No regular polyhedron bounded by equilateral triangles and having more than five at each vertex is possible, for six or more angles of 60° are equal to or exceed four right angles and cannot form a polyhedral angle.

The regular polyhedron next in order of simplicity to those formed by the equilateral triangle is the polyhedron formed by the square, each of whose angles is a right angle. Three right angles can be combined to form a polyhedral angle. It is seen then that a regular polyhedron can be formed, bounded by squares, three at each vertex. There is such a polyhedron. It has six faces and is called a cube or regular hexahedron. (Plate 2, fig. D.) No regular polyhedron bounded by squares and having more than three at each vertex is possible, for four or more right angles cannot form a polyhedral angle.

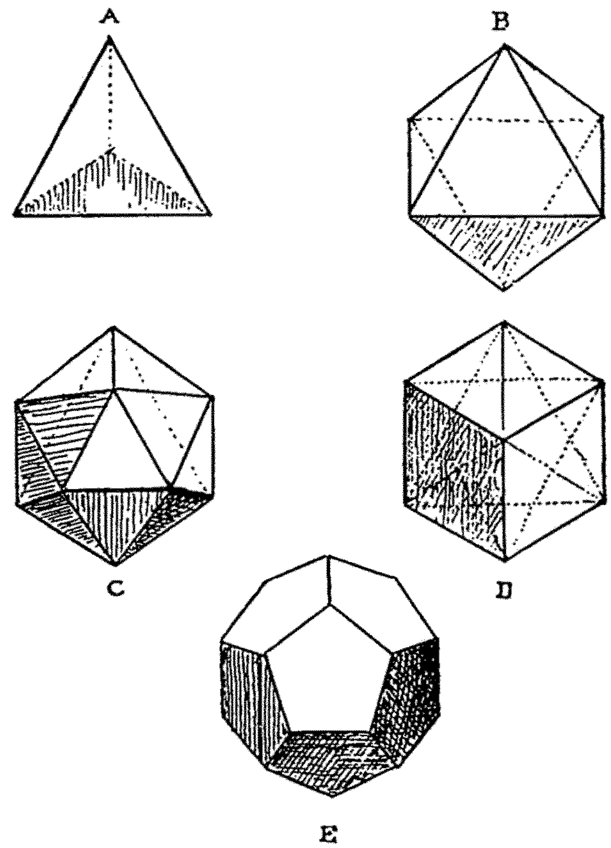


Plate No. 2—The Polyhedra

The next regular polyhedron is that formed by the use of the pentagon, each of whose angles contains 108°. Three angles of 108° each can be combined to form a polyhedral angle. It is seen then that a regular polyhedron can be formed, bounded by regular pentagons and having three at each vertex. There is such a regular polyhedron. It has twelve faces and is called the dodecahedron. (Plate 2, fig. E.) No regular polyhedron having more than three angles of 108° at each vertex is possible. These five polyhedra, the tetrahedron, the octahedron, the icosahedron, the hexahedron, and the dodecahedron are the only ones possible.

It would be well for the serious student to construct models of these five figures in order fully to comprehend their meaning. This can be done easily and with perfect accuracy by drawing on cardboard the following outlines, cutting them out and gluing the edges. (See Plate 3.) It may easily be seen by examination that the square, the pentagon, and the hexagon by its equilateral triangle are all that appear to the eye in regarding them. This trinity of forms is now disclosed to be harmonic with all five of the polyhedra and they are therefore all that are necessary in an analysis of proportional spaces, excepting the Egyptian triangle with the angles of 38° 30′ and 51° 30′ and the ideal angles of 42° and 48° which will be considered later.

An examination of the above system of angles, which proved of such value to the Greeks, may prove of equal value to the architects of our day, for by its use they can

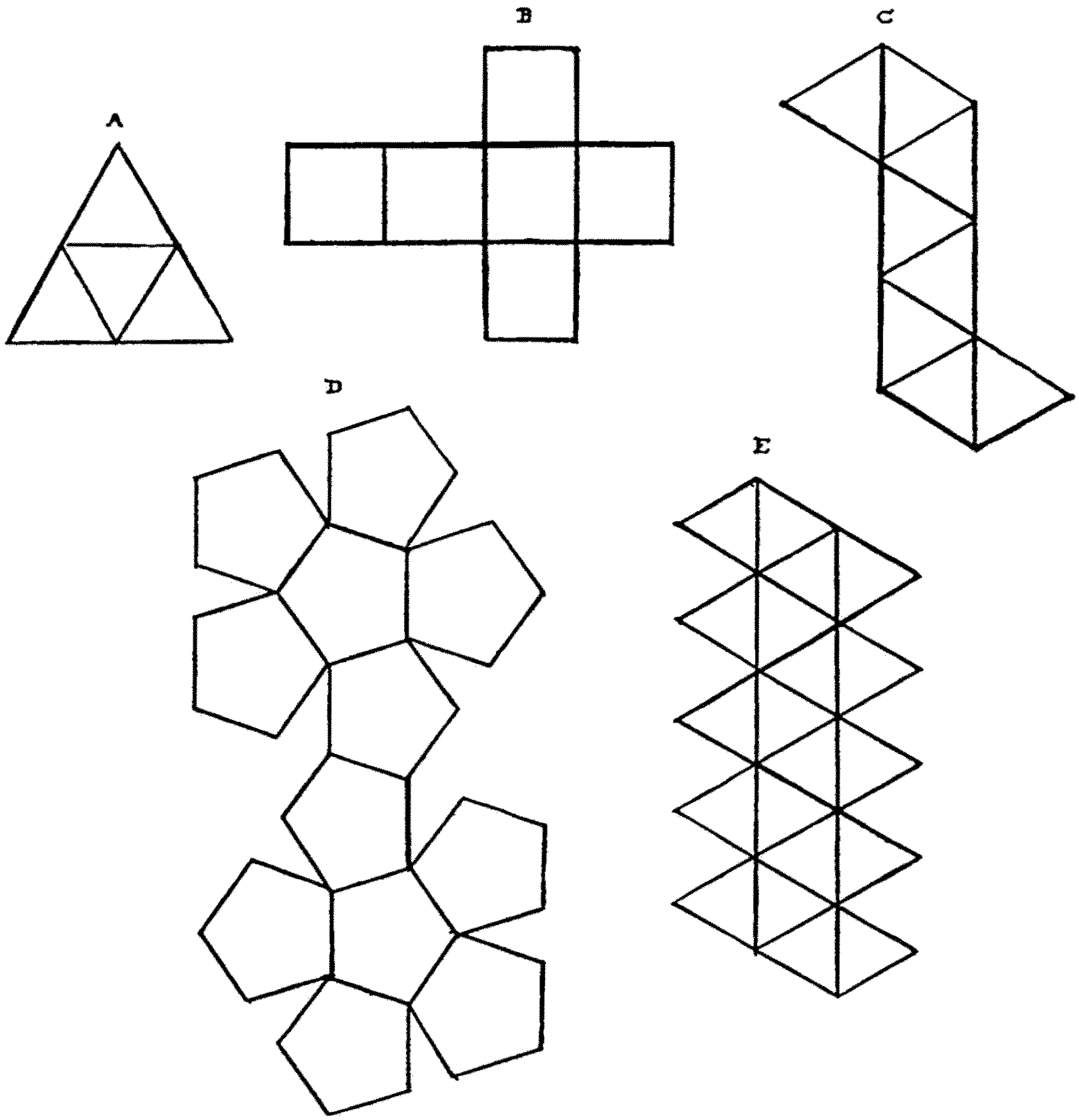


Plate 3. Polyhedral models

create a form-composition not only with greater ease but with a more perfect correlation of parts, which is one of the secrets of success in all forms of art. Moreover, the principle is capable of infinite combinations such as occur in snow crystals, for example, the marvellous forms of which are never repeated, although the angles of 30° and 60° are the basic elements of all their proportions. For the above reasons it is clear that the student of beauty can hardly waste the time which he expends in a close investigation of geometric relations, however valuable that time be to him, for these relations are the divine parents of proportional form.

Many of these principles of harmonic ratios were undoubtedly understood and made use of by the Free Masons, but have been lost or forgotten for nearly two centuries, as they were never allowed to be reduced to writing, but only handed down from master to apprentice as inviolate secrets. To-day few even of the fraternity know aught of the real significance of the ancient order or of the badges and emblems they wear, such as the seal of Solomon and numerous other designs at one time fraught with so much symbolic meaning. Investigators have been busy of late years in an endeavor to recover these lost secrets and the so-called "Divine Section" of Pythagoras has been set forth by some as the basic element of the principle, and many other theories have been advanced in relation to the subject. Books have been written and papers published in vain as the world still scoffs at what it considers the thoughts of madmen, stigmatizing their ideas as "mere cabala." These ideas contained the elements of truth, however, and if their authors had drawn their facts directly from Nature, giving scientific evidence for their deductions, they would in all probability have been accepted at once, for empirical statements, however plausible, can never be convincing.

In the light cast on the question of proportional growth in plants by the profound investigations of the most advanced modern botanists, we have tangible evidence to guide us and from what is thus revealed it would appear that Nature is forever striving to reach perfection through the use of an ideal angle; while extreme and mean ratio takes an even more important place among her methods. Six centuries ere the dawn of the Christian era Pythagoras promulgated theories in relation to geometry which were much discussed by his disciples; thus principles were formed which laid the foundations of geometric correlations applied by Hermogenes to architecture, and afterward employed by Ictinus in a still more intelligent way in the construction of his divine Parthenon. To-day the knowledge of the laws of growth in Nature is much more thorough among botanists than heretofore, but the result has only served to prove that the geometric principles advanced by Pythagoras have a fundamental influence on plant-growth, not only in relation to the octagon, the pentagon, and the hexagon but in an even more profound way in regard to extreme and mean ratio. Thus the "Divine Section" can now be shown to be one of the greatest factors for the just development of the proportional spaces in all flowers, plants, and shells, and so much is this the case that on the basis of the series of 5:8:13: etc., a theory of proportion may be advanced that will approximately supply the requirements of all students in the art of design.

A large part of the material herein produced is far from new, much of it dating back to the Egyptian priests from whom Pythagoras first learned his laws of geometry, in the proof of which we have the "Egyptian triangle" wherein the first elements of proportion have their being. It develops at once, by the natural progressions of its complementary angles of 51° 30′ and 38° 30′, the great series of harmonic ratios by the means of which Nature produces the perfect spacing of all parts of her beautiful forms.[1] What I have rendered as more or less new is an analysis of these forms as deduced from geometric principles, advancing no theory without referring at once to some well-known and definite object in Nature as proof.

The simplest and most direct way of producing the harmonic angles with their ratios is by means of the protractor of the quadrant. This remarkable instrument has been in use for the purposes of angular measurement from immemorial time, and although there is no mention of its discovery in history, it was in all probability the invention of the Chaldean Magi. It was certainly employed by the Egyptian priests in their examination of the heavens and in other early research, and it still remains in use for similar purposes, as nothing better can be devised. The circle, as we know, is arbitrarily divided, for purposes of computation, into 360 parts or degrees, as no other figure can be found to combine as a common multiple the lower integers 2, 3, 4, 5, 6, 8, and 9 in so perfect a way, together with many of the higher numbers as 10, 12, 15, 20, 30, and others. Three hundred and sixty is divided by 2 three times, by 3 twice, and by 5 once, as described in the following arrangement:

A.	2)360 2)180 2) 90 45	3)360 3)120 40	5)360 72
B.	3) 45 3) 15 5	5) 40 8	2)72 2)36 2)18 9
C.	5) 5 1	2) 8 2) 4 2) 2 1	3) 9 3) 3 1

The division of the quadrant of 90° by two produces the harmonic angles of 45°;

[1] See explanation of Egyptian triangle, Appendix Note E.—Editor.

if divided by four, the resultant angles are 22° 30′, 45°, and 67° 30′; if divided by eight, we have 11° 15′, 22° 30′, 33° 45′, 45°, etc.; and the division by three produces the angles of 30°, and 60°; dividing by six gives us 15°, 30°, 45°, 60°, and 75°; by division by five, we have 18°, 36°, 54°, 72°; division by ten renders the angles of 9°, 18°, 27°, 36°, 45°, 54°, 63°, 72°, 81°, and 90°, together with the supplementary angles of 99° and 108°. These constitute the angles of the octagon, the hexagon, and the pentagon, or all of those necessary in an analysis of proportional form with the exception of 42° and 48° of the ideal angle and the complementary angles of the Egyptian triangle, which will be taken up later on. By the use of these and the trinity of figures we have described, the divisional spaces of Nature's most beautiful works may be justly established.

A method will now be submitted for employing the quadrant by its protractor, which instrument should be constantly in the hand of all students of proportion.

It may first be stated, at the risk of a somewhat pedagogic effect, that the division of the circle by its diameter into two equal parts gives the base or horizontal line from which all angles are calculated by the degrees in the arc of 180° subtended by this diameter, 90° being half; and these degrees being divided into minutes, which in turn are divided into seconds. The vertical line drawn from 90° is a right angle with the base line and therefore the fundamental one for the establishment of all other angles. Plate No. 4 illustrates how the "protractor" of the quadrant "A" may be placed in relation to the circle "B," together rendering the relations of both sides of the angles, such as 30° on the one side and its complement of 60° on the other, for the

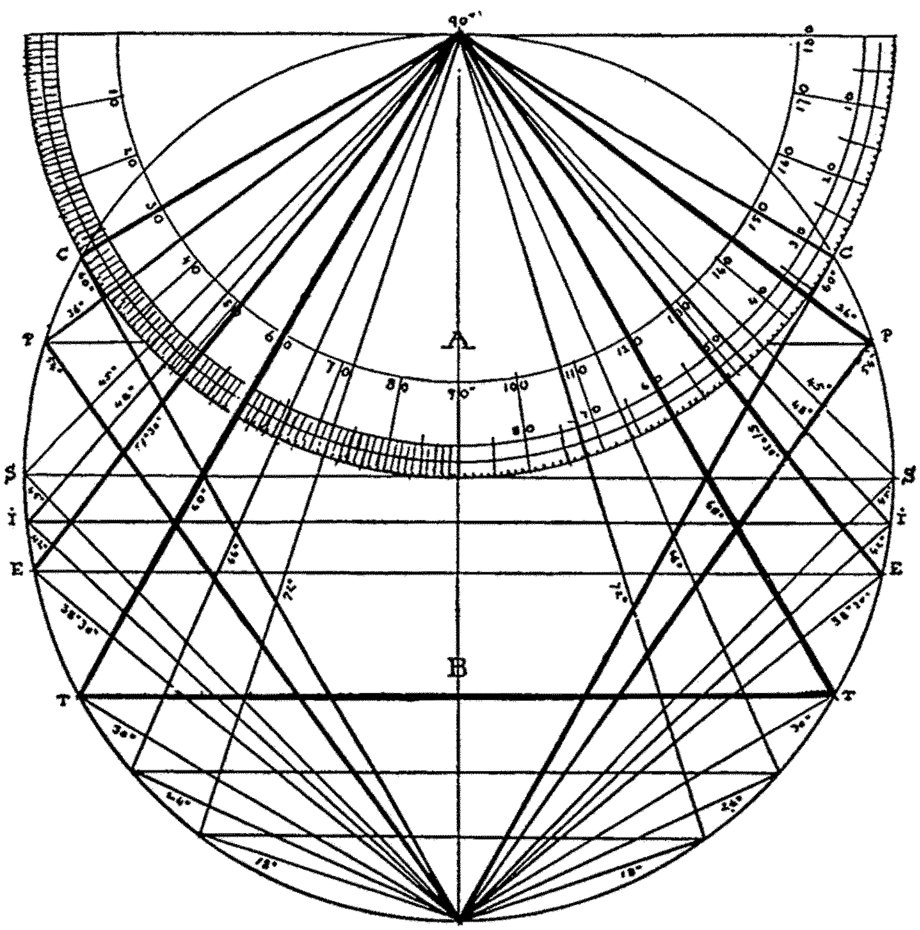


Plate 4—The Protractor

equilateral triangle; 36° and 54° for the pentagon; 38° 30′ and 51° 30′ for the Egyptian triangle; and 42° and 48° for the ideal angle: the angle of 45° coming on each side of the horizontal line at S. In other words, by ruling 30° from the pole or 90° in A, its complement of 60° will pass to the pole or 90° in the circle B and so on. In this chart only the angles of 30°/60°, 36°/54°, 45°, 42°/48°, 38° 30′/51° 30′, 24°/66° and 18°/72° are drawn as they are the most important ones in any system of proportional form.

In order to understand the unity of result between the workings of the Egyptian triangle, the pentagon, the octagon, and the hexagon and the angles of polar force, it will be necessary to know some at least of the attributes of the forms of which I shall have such frequent occasion to speak, and I regret that the complications of the designs in this system of analysis are at times necessarily so great as to require patient and careful attention from the reader; such patience and attention, indeed, that many of them will protest against this intricate division produced in the necessary diagrams. *But there is no other way to present the facts* if the student wishes to obtain a clear and intelligent understanding of the unity of Nature and to comprehend the exact harmony of parts in her form compositions which establish this unity and which result directly from geometric correlations, revealing truth after truth. By their intersecting lines these harmonic progressions produce points for measurement, often considerably removed and which would never be suspected or recognized by the eye alone however greatly the mind may feel the resultant harmony. It should here be noted that no intersection of less than three lines can be accounted of any value, but the moment a third line crosses a given point, the result is all important; if four or more unite, still further importance attaches. The study of these details is then necessary if the student desires a knowledge of the unity of the pentagon, the hexagon, and the octagon which Nature so continually employs in conjunction. Moreover, they clearly explain why the great architects of antiquity selected these polygons as fundamental elements in the composition of their temples, churches, and other buildings; but more than all else they proclaim that "Order is heaven's first law," revealing in a large measure the hand of Divinity.

After a careful and intelligent study of these principles and their relations to each other, the eye will come to realize that geometric forms are really beautiful in the light cast upon them by the mind, which illuminates them until it throbs with unwonted satisfaction when any one of the polyhedra is regarded. But that mind must be awake; when it does not thus respond, it may be safely counted asleep or dead to the underlying glories of the Creator's constructive laws.

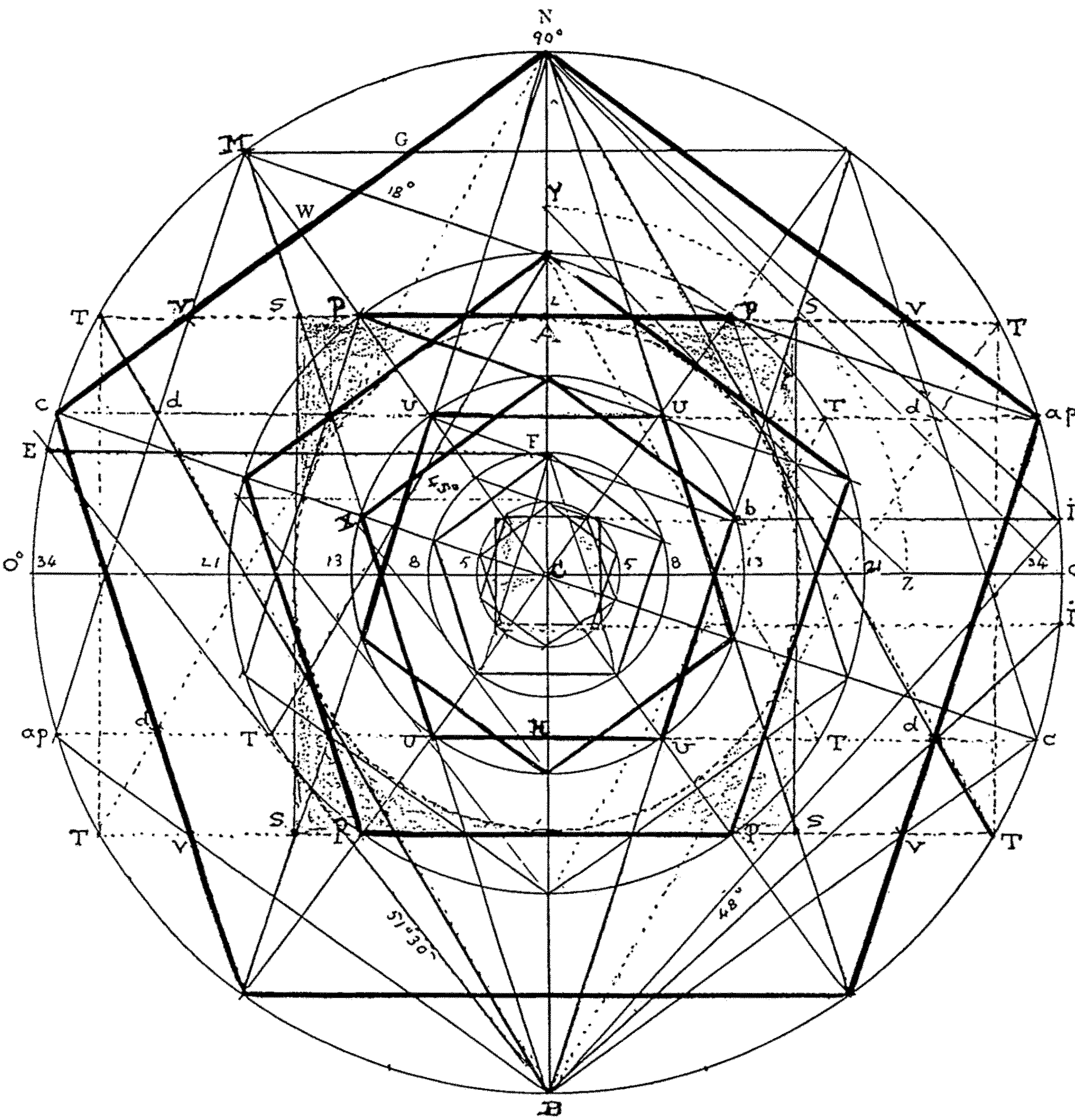


Plate 6.—The Pentagon and Square

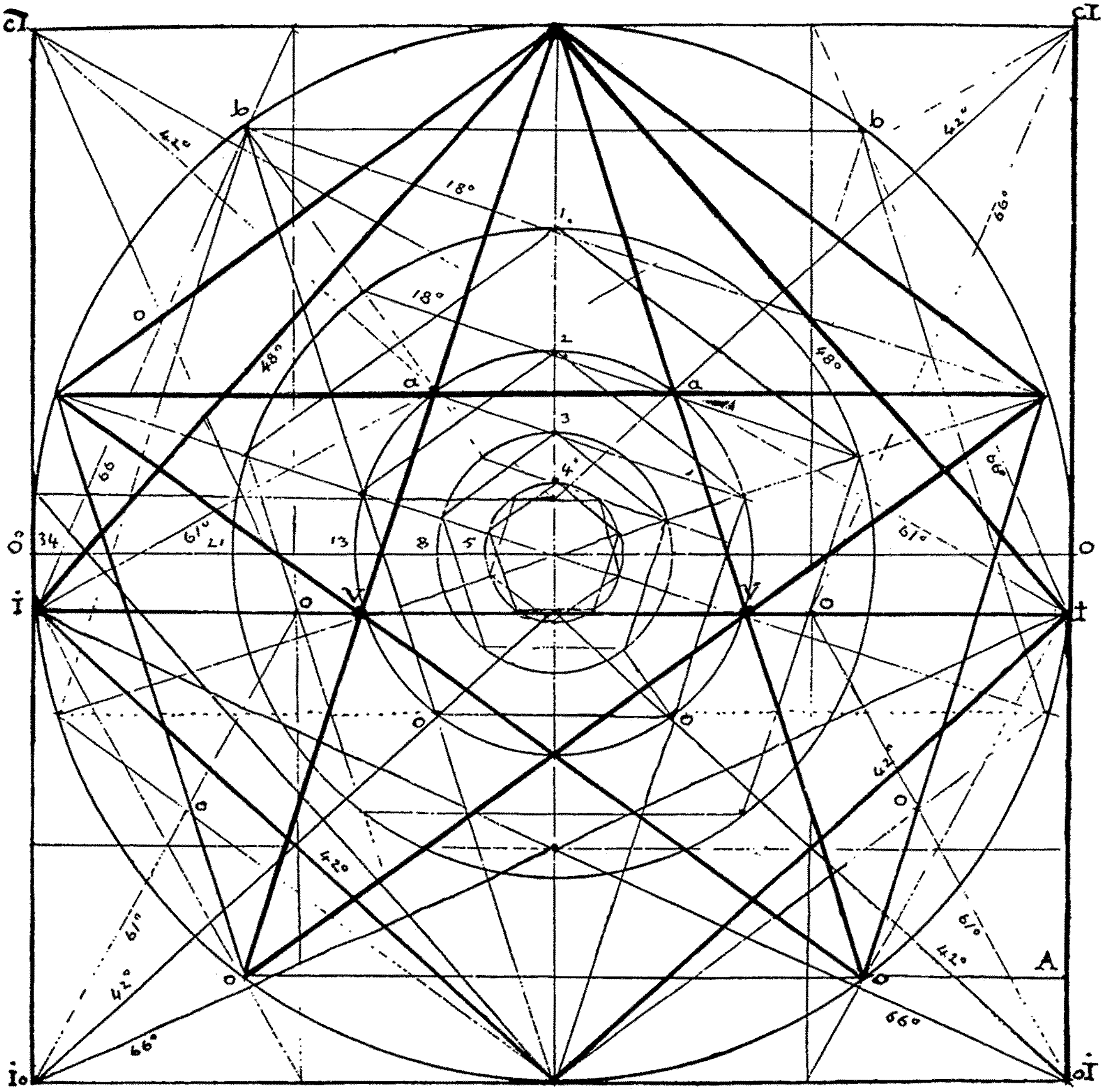
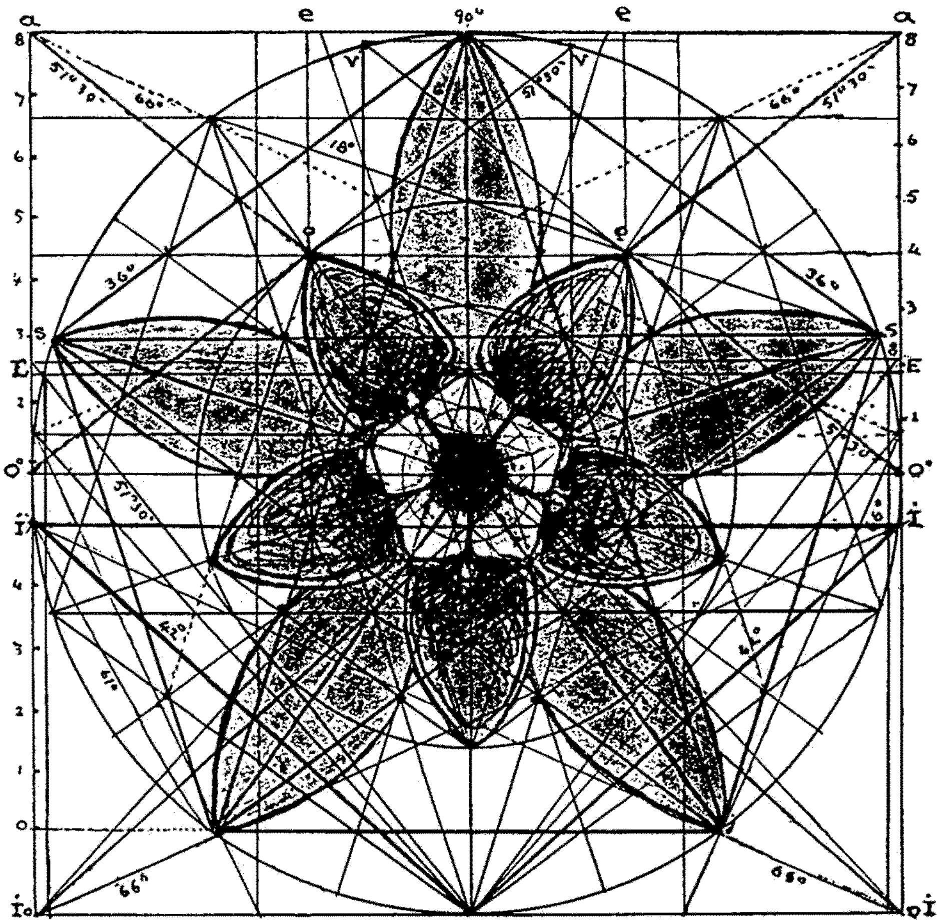
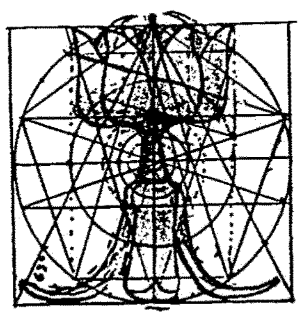


Plate 9. The Pentagram



A



B

Plate 10. Milkweed

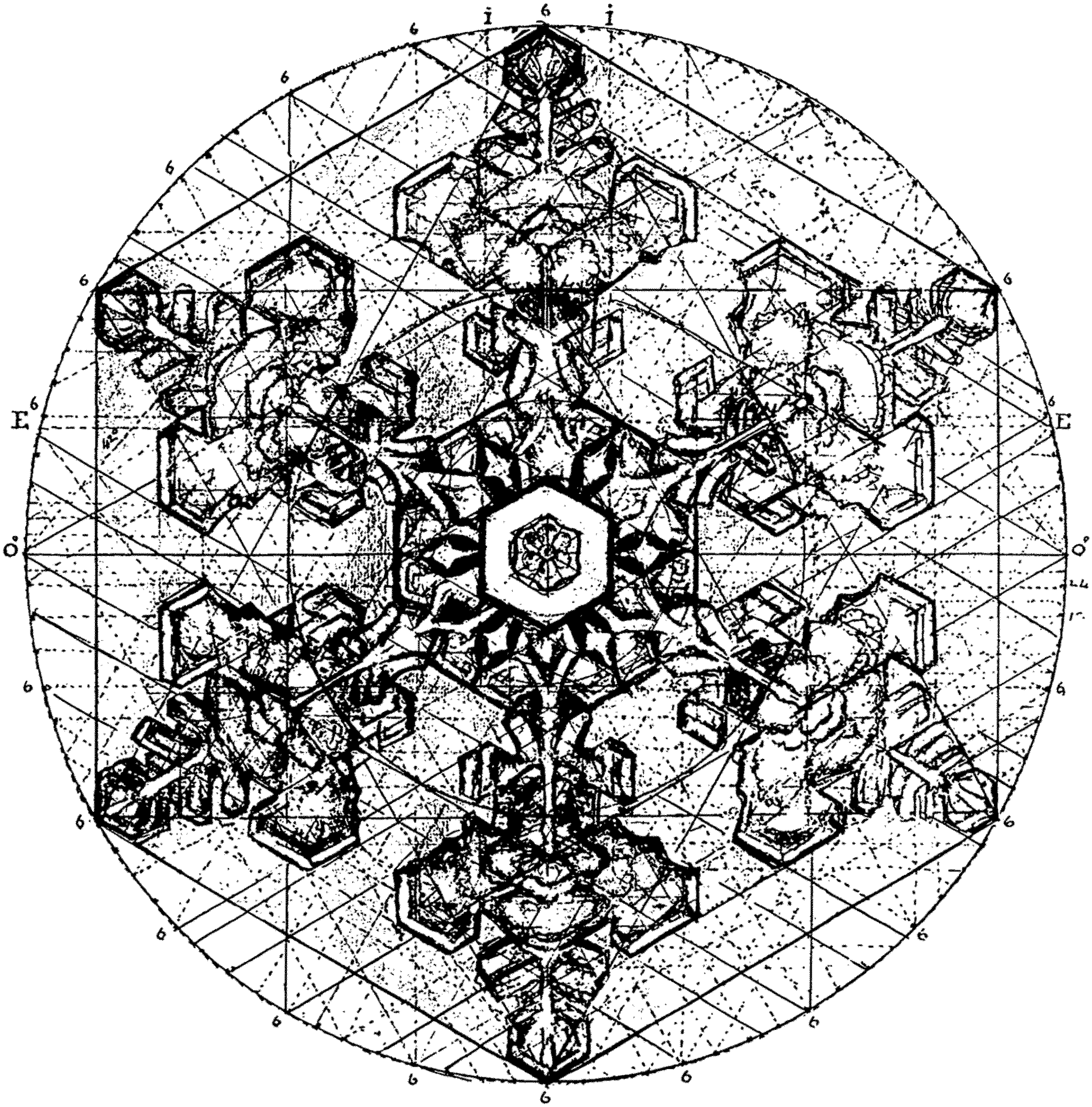


Plate 51—Snow Crystal

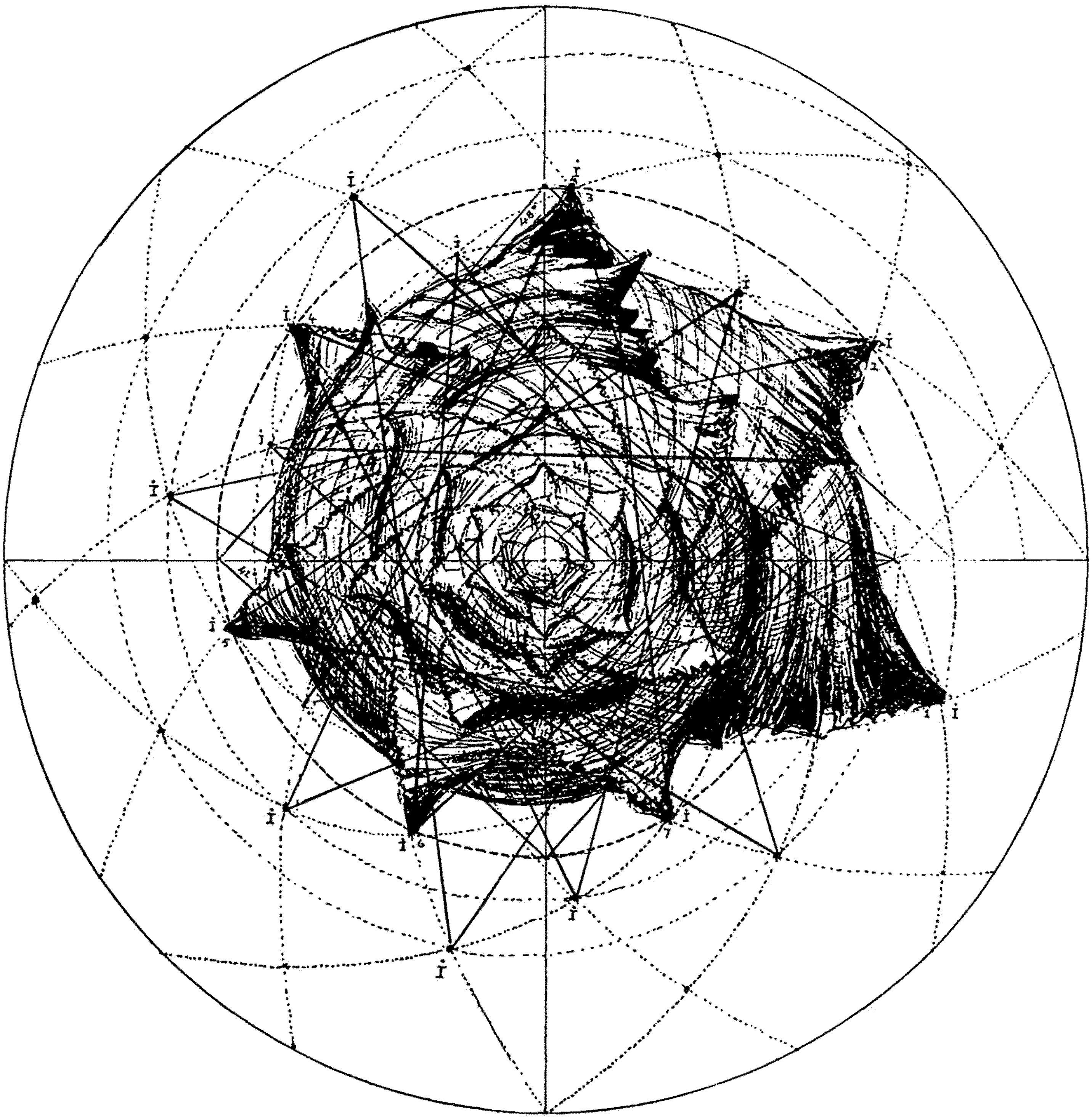
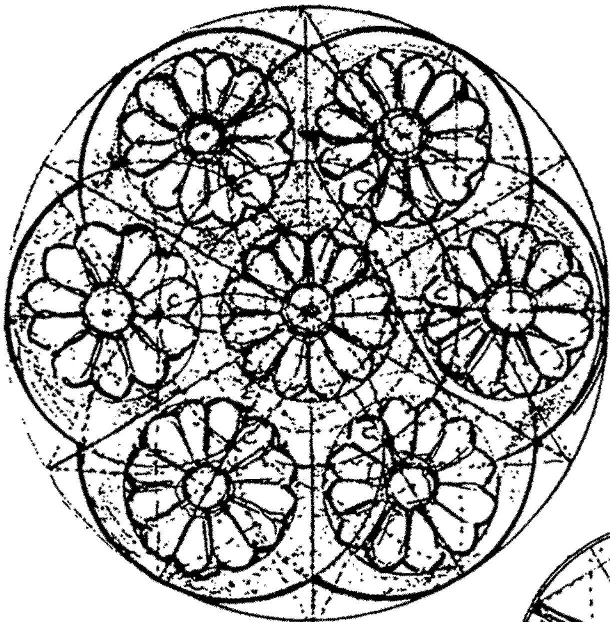


Plate 106—Plan of Murex



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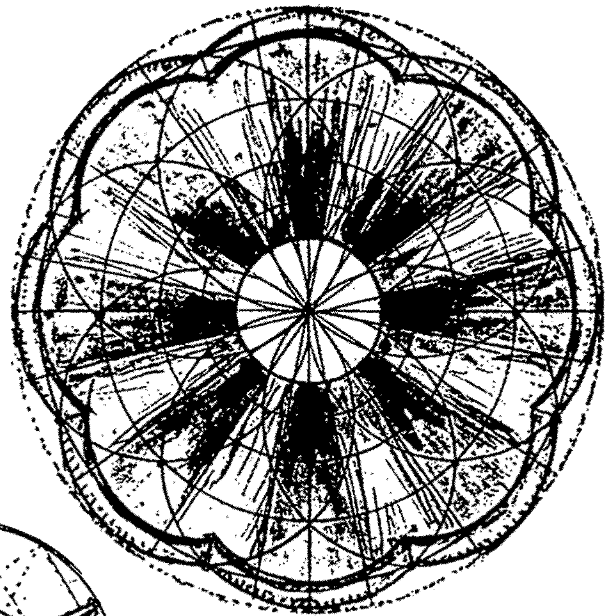

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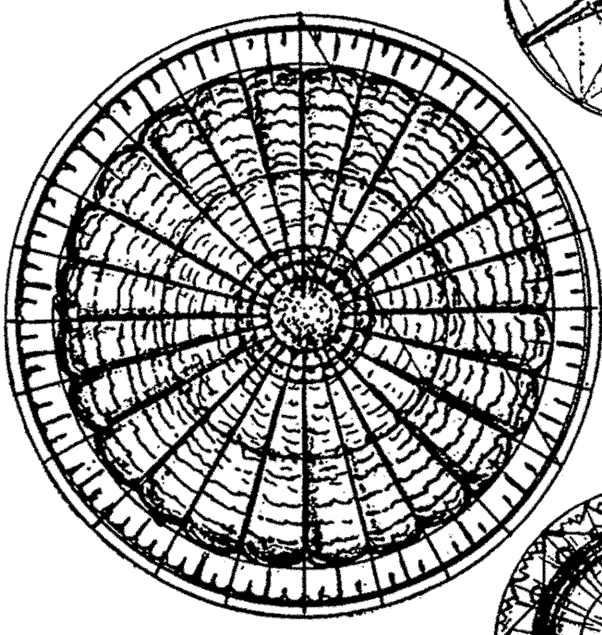

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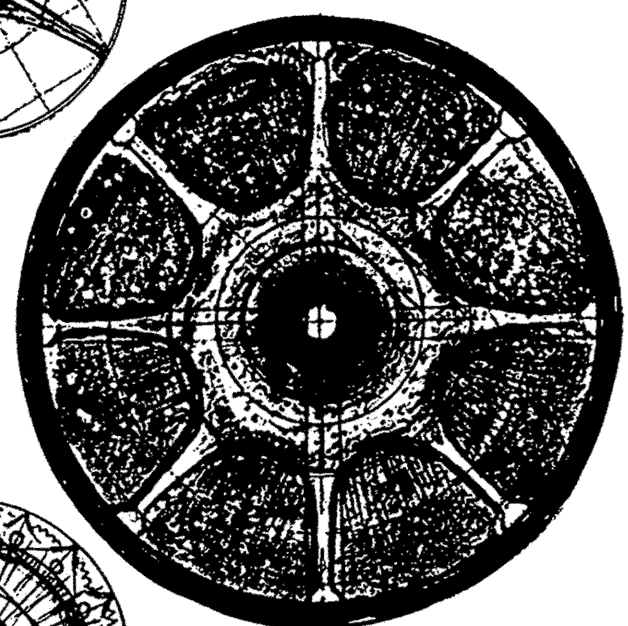

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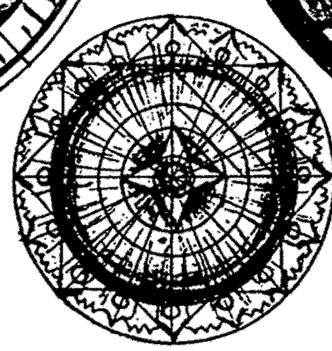

i

Plate 127—Diatoms

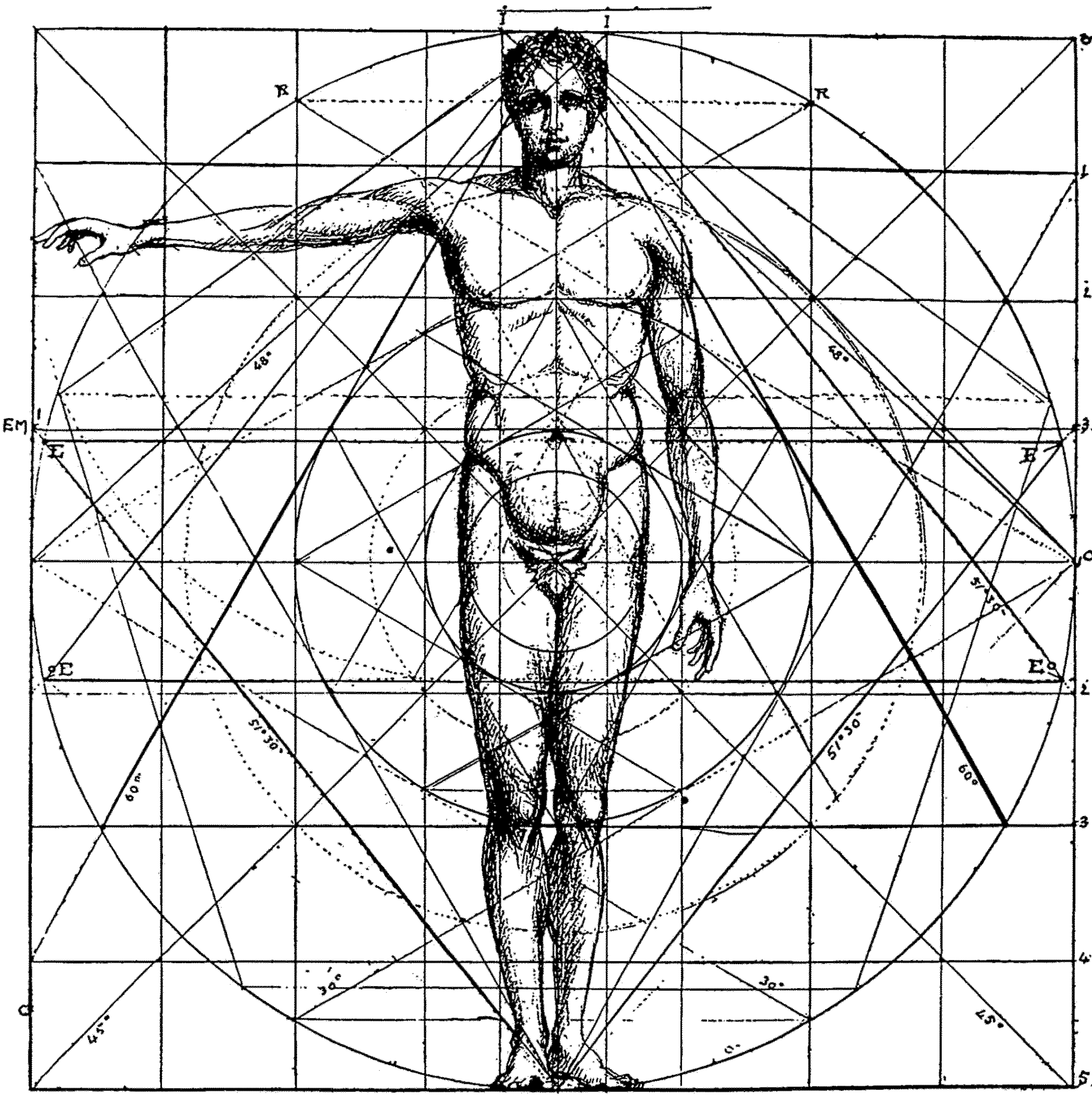

Plate 166—Figure of a Young Man

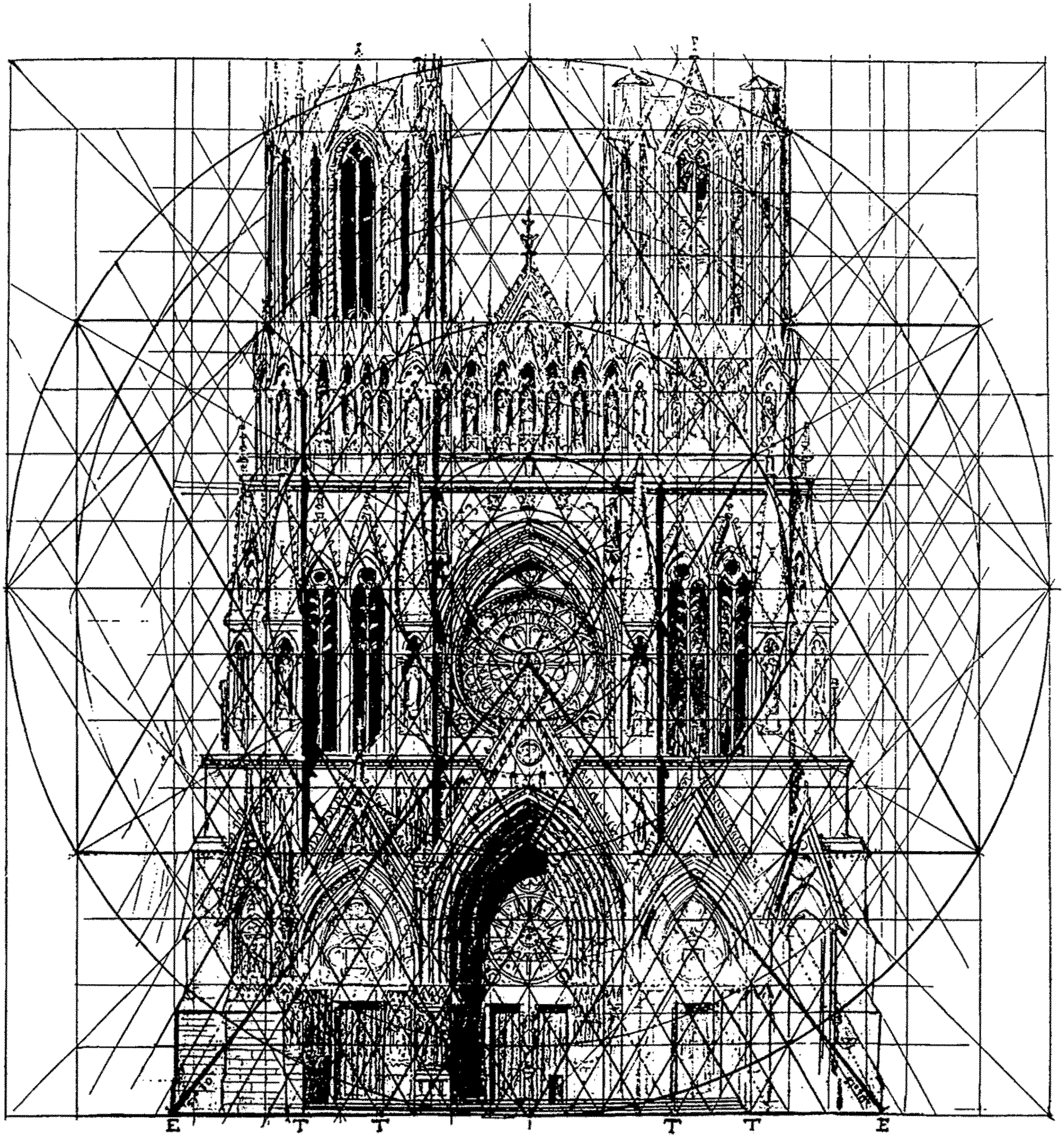

Plate 229—Rheims Cathedral

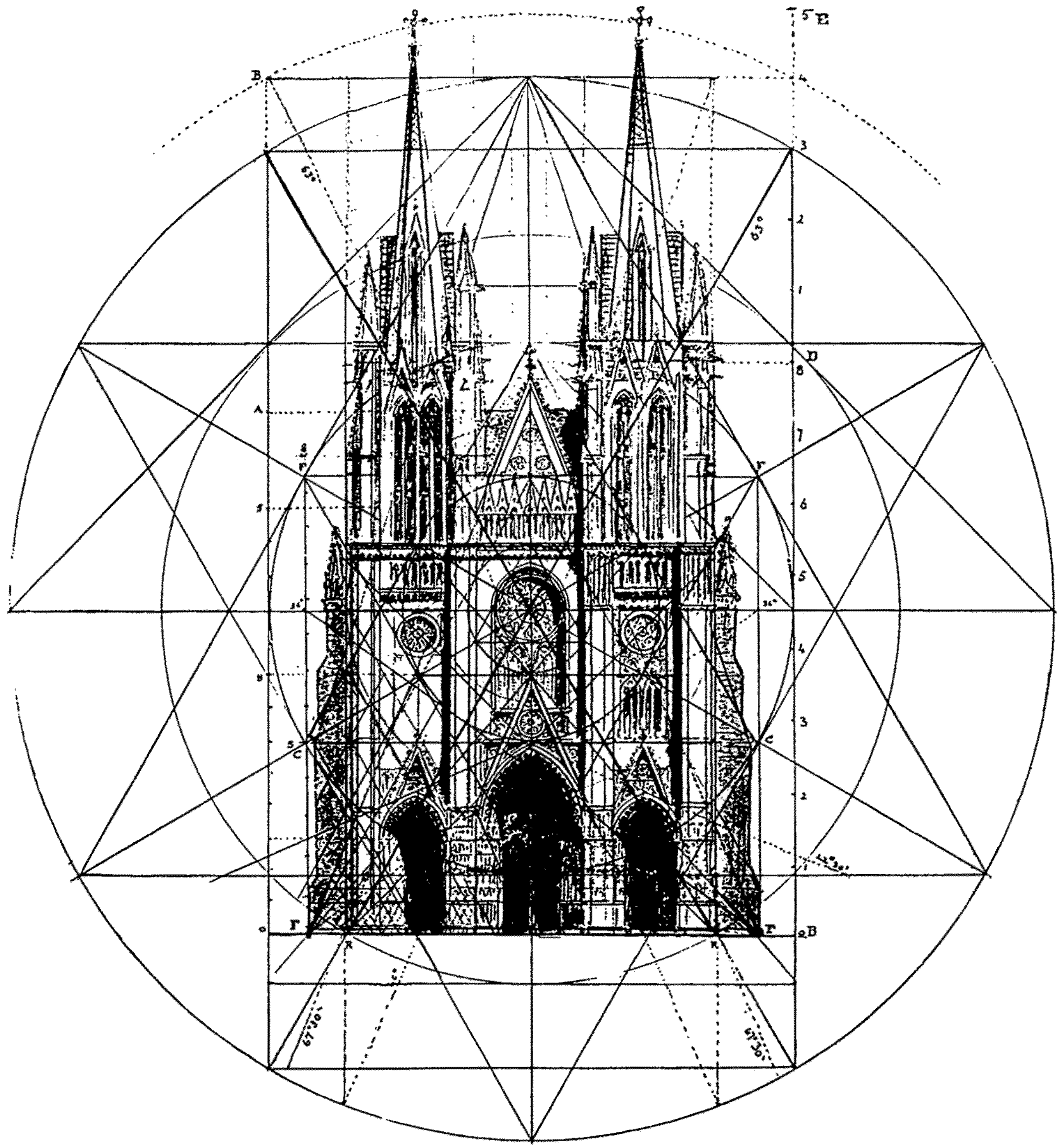

Plate 244—Design for Façade of a Cathedral