BEHOLD!

THE GRAND PROBLEM

THE

CIRCLE

SQUARED

BEYOND

REFUTATION

NO LONGER UNSOLVED

Mathematical
and
Geometrical Demonstrations

By

Carl Theodore Heisel

Disproving Numerous Theorems, Problems, Postulates, Corolleries, Axioms, and Propositions, with Ratios, Laws and Rules Hitherto Unknown in Mathematical and Geometrical Science, Naturally Growing Out of the

Extraordinary and Significant Discoveries of a

LACKING LINK

by Carl Theodore Faber

Who was a Citizen of Brooklyn, N. Y., U. S. A.

IN THE DEMONSTRATION OF THE WORLD RENOWNED

PYTHAGOREAN PROBLEM

Utterly Disproving Its Absolute Truth, Although Demonstrated as such for Twenty-Four Centuries; and by This Discovery Establishing the Fact of the Existence of Perfect Harmony Between Arithmetic and Geometry as a Law of Nature, and Calculating to Settle Forever the Famous Dispute Between

The Two Great Philosophical Schools

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A Citizen of Cleveland, Ohio, U. S. A.

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ADDENDA

Preface

To square the circle is to prove the exact numerical length of the circumference of a circle of a known diameter and the exact length of the side of a square of equal area to that circle.

For many centuries all of the greatest mathematicians have failed to solve the problem and for that reason the general belief today is that it cannot be solved. But such a conclusion does not justly follow. It only proves that up to the time of Carl Theodore Faber, it had not been solved.

The British and French Scientific Societies, over a hundred years ago, refused to entertain any further solutions of the problem, and the mathematical and geometrical profession of today, with few exceptions, are so hidebound and prejudiced in that belief that they even ridicule the idea of a solution for it and they will neither listen to a true solution when presented to them, nor attempt to disprove it. We freely acknowledge that the present approximate ratio of diameter to circumference and approximate circle area are near enough for all practical purposes, and no scientist, or any one else, has ever claimed that they are anything else but approximations. They are both infinite quantities and, therefore, cannot be exact, true, or accurate. How can it be possible for a square to have an infinite root? Can such a thing be sound sense? Any child knows that a circumference has, and of necessity must have, an exact length, and that every circle must have an exact area, and that they are not so much plus, or so much and a little more.

Many prominent mathematicians in past history have demonstrated fallacies and inconsistencies in the science of mathematics and geometry as taught today. Burkeley, conscious of many inconsistencies in the science of mathematics, proposed numerous reforms. The great Newton expressed his doubt in the existence of the indivisible in nature as not in harmony with his reason. Mathematicians of the calibre of Piscal and Legendre have irrefutably demonstrated the fallacy of the doctrine of a square being divisible into two equal squares, and yet modern science accepts the proposition that a circumscribed square is equal to twice the inscribed square as absolute truth.

A right-angle triangle may have two equal sides and it is proved by algebra and geometry that the two equal squares are visually and geometrically equal to a third square, but it cannot be proved numerically, and a numerical proof is the only absolute proof, for the beauty of it all is, that figures do not lie. Mathematicians may be divided into two classes, the philosophical and simply practical, or spiritual and material. The first have a knowledge or some conception of first mathematical principles, but the second have none whatever.

As a class modern mathematicians do not seem to possess any doubt. They dò not question anything. They blindly accept what is taught in the books as absolute and final. They learn their mathematics as a parrot learns language by simple imitation.

The 47th Problem of Euclid is true when the sum of the squares of two sides happen to equal a third square, but that is the exception and not the rule, for the sum of all other two squares form irrational quantities, which are not and cannot be squares, and have no square roots.

A square then, minus one unit is not a square number, and cannot be a square. It is an irrational number equal to $a^2 + 2ab + b^2 - b^2$, or $a^2 + 2ab$ forever. It is equal to a rectangular parallelogram. Therefore, such a parallelogram containing an irrational quantity cannot be equal to a square.

This formula, $a^2 + 2ab$, for an irrational quantity is the very basis of our discoveries. b^2 is the eternal difference between an irrational quantity and a square, however infinitesimal the square unit may be assumed. An irrational number is a number, from which no finite square root can be extracted, because it is not a square. Any claim to a solution of the mystery of "square with infinite roots," equal to a paradox, will, of course, have to be so clear in its demonstration, as to brand any doubting Thomas of science with blindness and stupidity. Truth is an irresistible force and will prevail. Accept the inevitable with good grace.

To some mathematics has no other significance to them than as it is applicable to mechanical use, while to others it is the "Science of all Created Things," as the great philosopher Liebnitz called it. This class has recognized the truth of our discoveries almost at sight. We rejoice in numerous converts, and among them men of scientific eminence. We have yet to meet with the first attempt at a refutation by actual numerical demonstration of our doctrine, although we have repeatedly tried to *provoke* attack by public call.

Hence, we invite readers with boldness of confidence to our claim of discovery.

Mr. Carl Theodore Faber himself was filled with wonder that this simple truth had not been discovered long before. He discovered the insidious cause by which even Euclid was misled in trying to carry it out practically. While Euclid had a correct insight, theoretically demonstrating even the logical necessity of the circle area being equal to a square, he was unable to prove it numerically.

This insidious cause resided in a false assumption of universal validity of the famous "Pythagorean Problem," finally found to be only a special one. The long prevailing ignorance of *"science"* of the fact that the so-called *Irrational Quantity* can ever be only equal to $a^2 + 2ab$, had a great deal to do with the difficulty of squaring the circle. When we found the truth, we found that truth is harmony, and that harmony is order and that order is the law of the universe.

We are discovering new ratios, laws and relations every day in every branch of science, that have never been known before, and we are discarding laws and rules that have been accepted as absolute truth in the past.

CHAPTER I

The Grand Problem No Longer Unsolved

Brief and Infallible Method of Squaring the Circle Discovered by Mr. Theodore Faber, of Brooklyn, N. Y., and Formerly a Merchant of Cleveland, Ohio, Published in the Brooklyn (N. Y.) *Eagle* February 28, 1883.

To affirm something in direct opposition to the publicly declared doctrine of such high authorities as those of the French Academy of Science and the British Royal Society, obviously needs the support of infinitely more powerful proof than these two great authorities are enabled to furnish in support of their dictum of impossibility of the circle-quadrature. More than twenty-four centuries of failure on the part of science to solve the problem, after an intense search, especially on the part of astronomers who, in its solution, discerned the only possible guarantee as test for their calculations (even though now it be solved beyond all power of refutation) certainly seem to constitute a plausible foundation for such dictum, although not defensible in a strictly scientific point of view, for such dictum found itself based on no other ground than the fact of its still remaining unsolved, especially in the face of Euclid's demonstration of the logical necessity of the solution of the problem, which he himself, however, failed to solve numerically—a failure which, together with all the failures after his own, Mr. Faber finally discerned to be solely due to a most insidious cause residing in a false assumption of universal validity, of the famous "Pythagorean Problem," finally proved to be only a special one! The remarkable discovery of this fact by Mr. Faber at once destroys all logical basis for the assumption of an infinite root, and thereby frees the solution of the grand problem from all further difficulty. Nothing, in fact, remains but to give the discoverer's two rules for squaring the circle, by the correct application of which any fair and candid devotee of the science can satisfy himself instantly of the perfectly incontrovertible truth of the solution of the problem, viz.:

Rule 1. Convert any given circle-diameter by the ratio nine to eight (9:8) and you have the exact square-root of the circle area.

Rule 2. Multiply any given circle-diameter by the mixed number three and thirteen eighty-firsts ($3\frac{13}{81}$) and you have the circumference; half of which, multiplied by radius, gives the circle-area (according to Euclid).

The correct application of these two rules affords their own mutual test, or criterion of the truth of the respective two ratios employed, since whatever number may be assumed for circle-diameter, this invariable result ensues, namely: absolute coincidence of square-area, as produced by the respective two ratios: 9 :8 and 1 :$3\frac{13}{81}$, thereby furnishing incontrovertible proof of the exact truth of both the respective ratios.

RULE 1—TABLE OF PROOF

Conversion Ratio	Circle Diameter	Square Root of Circle Area		Circle Area
9 : 8	1	$\frac{8}{9}^2$	=	$\frac{64}{81}$
	2	$1\frac{7}{9}^2$	=	$3\frac{13}{81}$
	3	$2\frac{6}{9}^2$	=	$7\frac{9}{81}$
	4	$3\frac{5}{9}^2$	=	$12\frac{52}{81}$
	5	$4\frac{4}{9}^2$	=	$19\frac{61}{81}$
	6	$5\frac{3}{9}^2$	=	$28\frac{36}{81}$
	7	$6\frac{2}{9}^2$	=	$38\frac{58}{81}$
	8	$7\frac{1}{9}^2$	=	$50\frac{46}{81}$
	9	8^2	=	64
	10	$8\frac{8}{9}^2$	=	$79\frac{1}{81}$

RULE 2—TABLE OF PROOF

Ratio Between Diameter and Circumference	Circle Diameter	Circumference	Half Circumference		Radius		Circle Area
1 : $3\frac{13}{81}$	1	$3\frac{13}{81}$	$1\frac{47}{81}$	×	½	=	$\frac{64}{81}$
	2	$6\frac{26}{81}$	$3\frac{13}{81}$	×	1	=	$3\frac{13}{81}$
	3	$9\frac{39}{81}$	$4\frac{60}{81}$	×	1½	=	$7\frac{9}{81}$
	4	$12\frac{52}{81}$	$6\frac{26}{81}$	×	2	=	$12\frac{52}{81}$
	5	$15\frac{65}{81}$	$7\frac{73}{81}$	×	2½	=	$19\frac{61}{81}$
	6	$18\frac{78}{81}$	$9\frac{39}{81}$	×	3	=	$28\frac{36}{81}$
	7	$22\frac{10}{81}$	$11\frac{5}{81}$	×	3½	=	$38\frac{58}{81}$
	8	$25\frac{23}{81}$	$12\frac{52}{81}$	×	4	=	$50\frac{46}{81}$
	9	$28\frac{36}{81}$	$14\frac{18}{81}$	×	4½	=	64
	10	$31\frac{49}{81}$	$15\frac{65}{81}$	×	5	=	$79\frac{1}{81}$

If the square of circle-diameter be equal to $a^2 + 2ab + b^2$, the circle area will *invariably* constitute a^2 of that formula, while twice the square root found by Rule 1, multiplied by the difference between the given circle-diameter and the square root found by said rule, is ever = 2ab, and the square of the difference is = b^2.

Assume any circle-diameter you please, and the correct application of the rule will always confirm this law.

Let us, for example, use the circle-diameter = 100, then as 9 :8 :: 100 :$88\frac{8}{9}$ = square root of the area of a circle of diameter 100.

$$\text{Now } 88\tfrac{8}{9}^2 = 7901\tfrac{19}{81} = a^2$$
$$2 \times 88\tfrac{8}{9} \times 11\tfrac{1}{9} = 1975\tfrac{25}{81} = 2ab$$
$$11\tfrac{1}{9}^2 = 123\tfrac{37}{81} = b^2$$
$$\overline{10{,}000} = 100^2$$

If we take "9" for circle-diameter, the square root of the circle-area is $=$ 8.

$$
\begin{aligned}
8^2 &= 64 = a^2 \\
2 \times 8 \times 1 &= 16 = 2ab \\
1^2 &= 1 = b^2 \\
&\overline{81} = 9^2
\end{aligned}
$$

This *absolutely proving the quadrature of the Circle.*

What mental effort it has cost for upwards of a quarter of a century to complete this great discovery, in vain sought after for so many ages past, cannot easily be conceived.

The final discovery of *so great a truth* cannot be otherwise than beneficent—only remember that already 707 decimals had been added to the orthodox infinite ratio, with the object of reaching an *approximate circle-area;* while Faber, by the simple ratio 9 :8 obtains the area *perfectly* without the use of any decimals at all! What saving of time and labor!

The circumference of a circle of diameter $=$ 1, is $=$ area of a circle of diameter 2—what is the area according to our 9 :8 ratio?

Answer: $9:8 :: 2:1\frac{7}{9}$ and $1\frac{7}{9}^2 = 3\frac{13}{81}$.

This, then, is the *true* relation between circle-diameter and circle-circumference, namely, $1:3\frac{13}{81}$, which is itself equal to *a square* as seen above.

The fact that over 100 years ago two of the greatest scientific societies in Europe publicly announced *their belief* that the solution of the "Grand Problem" of the circle quadrature is *impossible* will only serve largely to enhance the credit due to this great American discovery, which cannot fail to be *immortal,* because absolutely irrefutable!

CHAPTER II

A Demonstration of a New Law in Geometry—A Lacking Link Leading to the Solution of Unsolved Problems

Our primary object is to develop the *new law* in as simple language as possible compatible with comprehension, and incidentally to demonstrate the two most salient effects of the new law, viz.:

First—In proving the Pythagorean Problem, assumed as a universal one to be rank fallacy.

Second—In establishing irrefutably the only true ratio between the diameter and circumference of a circle.

To philosophers conscious of the vast difference between approximate and absolute truth, the great significance of the discovery need not be pointed out. The new law reads as follows:

An eternal difference exists between a square and a so-called irrational quantity.

The attention of readers familiar with geometry needs no particular call to the fact that this new doctrine is in diametrical opposition to the long-taught doctrine of Pythagoras, inculcating as truth the existence of "a square with an infinite root," which notion actually furnished the very impulse to the discovery of the new law, by creating an intense skepticism in the mind of Mr. Theodore Faber as to the truth of this remarkable doctrine, followed, after a decade of vain search for the seat of error, by the final revelation of the truth.

An infinite root must strike every reflective mind, as utterly incompatible with a conception of form, the very element of geometry, a science which can have no possible meaning in the formless, in which there can be no difference.

A so-called "hypothenuse," then, must be a finite quantity. This necessity led to the final discovery of an artificial root of an irrational quantity, with the following rule for determining the same, viz.:

First—From any given irrational quantity extract the largest square, calling the root of the same $=$ a.

Second—Dividing the difference between the given quantity and the square by twice a, calling this fraction $=$ b. This a $+$ b root, when squared, is equal given irrational quantity, $+ b^2$, therefore the irrational quantity is $= a^2 + 2ab$ forever. The new law is simply a deduction from this rule, and defies all science to upset it.

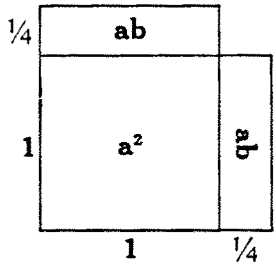


FIGURE No. 1

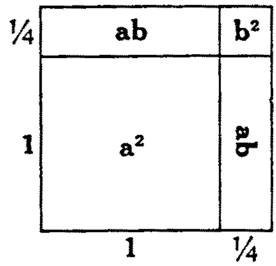


FIGURE No. 2

It requires but a slight reflection to discern that this law is an instant subversion of the Pythagorean problem, as a universal problem, and that consequently all astronomical calculations made by aid of the same are radically erroneous. And since the new law affects the proposition upon which the famous ratio 3.14159+ is based, that ratio is fallacious, and by no means possesses the infallibility ascribed to it by our modern authorities.

What, then, is the true ratio? It is a question of much deeper significance than is generally imagined, since it is pregnant with revolution to both science and philosophy.

The second salient effect of the new law was stated as consisting in the determination of a true ratio between the diameter of a circle and its circumference, equivalent to the quadrature of a circle, hitherto considered as utterly incapable of proof, but its discovery furnishes at once the fact of its being as capable of proof as any geometrical problem, provided the evidence be fully considered so that the conversion square appears as an indisputable fact before the eye. For this purpose it is needful for the party desirous of mastering the problem to apply the closest attention to the following directions, given for the construction of a diagram which may fitly be called fundamental in the solution of

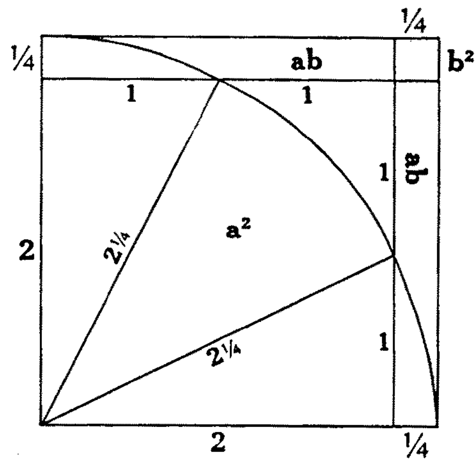


FIGURE No. 3.

the problem, and in which the use of algebraic symbols is purposely avoided, as a hindrance rather than an aid to demonstration, viz.:

First—Construct a square.

Second—From the middle of the summit side to the opposite angle, draw a straight line, dividing the square into two unequal parts, the smaller forming a right-angled triangle, the sides of which are related to one another as 2:1.

Third—Using the hypothenuse of this right-angled triangle for a radius, describe an arc towards the left until the radius falls in line with the perpendicular side of the square.

Fourth—Moving the same radius to the right from the middle of the summit side of the square until it falls in line with the base of the square, the two combined arcs forming the quadrant.

Fifth—Complement now the square of the radius, circumscribing the quadrant.

On closely contemplating this diagram it soon becomes evident that the relation of the radius to the side of the square is similar to the relation of the diameter to the square root of the circle area, and that, if we can determine the height of the arc, equal difference between radius and side of square, we should at once possess the true ratio, from which that between diameter and circumference is deducible.

Now how are we to determine the height of the arc? Knowing that this sought quantity is equal to difference between radius and side of square, and that the radius is equal to hypothenuse of the right angle triangle, whose sides are as 2:1, all we have to do is to determine that hypothenuse.

According to the system of Pythagoras, it would be equal to the infinite decimal root of the sum of the two squares of the right angle 2:1, hence equal to 2.236+, while according to the new law, which repudiates decimals, it would be equal to the largest square in 5, plus the difference between 5 and 4 divided by twice 2, together equal to 2¼. Therefore, the ratio between the radius and side of the square is equal to 2¼:2, equal to 9:8. The ratio 9:8 is a more simple expression than that of 2¼:2, which latter was found by analysis of the primary right angle 2:1. This is the identical ratio between the diameter of a circle and square root of its area, from which we can easily deduce that between the diameter and the circumference. For instance, assuming a diameter of 9, the area would be equal to 8^2; the theory of the area of a circle being equal to the product of half radius and circumference holds good; therefore, all we have to do is to divide 64 by half radius (2¼) to obtain the circumference equal to 28⅘, and this, divided by 9, gives $3\frac{13}{81}$ for circumference of a circle whose diameter is 1, thus establishing the true ratio.

A collateral self-evident proof of the correctness of the ratio 9:8 is furnished by a diagram in which two other ratios, namely 10:9 and 8:7, are applied with that of 9:8; so as to show their respective effects upon the quantity cut off from a^2 by arc on one angle and quantity held by the arc above the side, from which it is evident that the equilibrium between those two quantities is instantly destroyed, if either of the two other ratios be applied, showing that the medium ratio 9:8 is the only true one. (See Figure No. 4.)

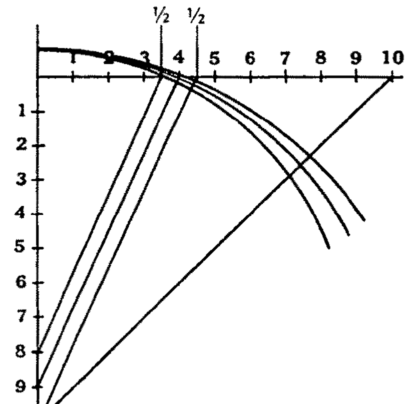


FIGURE NO. 4

The author would feel like committing a blunder if he neglected to call special attention to a peculiar property of the all important and sole integral right angle triangle, which can be inscribed in a circle or a quadrant, with sides related to one another as 2:1, as elucidating some facts in the law of geometrical progression in circles, the principle of which has scarcely been understood. For instance, why should the area of one circle be equal to the circumference of another, as in the case with two circles whose diameters are 2 and 1? And again, why should the area of one circle be equal to the circumference of the same circle, as in the case with the circle whose diameter is 4? The true ratio, $1:3\frac{13}{81}$, is deducible from this very law of geometrical progression of circles.

The peculiar property lies in a three-fold possibility of evolution in our triangles, viz.:

First—Revolving this right angle triangle around its larger side, for an axis, the circle described has a radius equal to 1 and a diameter equal to 2.

Second—Revolving the same triangle around its hypothenuse for an axis, the circle described has a radius equal to $\frac{8}{9}$ and a diameter equal to $1\frac{7}{9}$.

Third—Revolving the same triangle around its smaller side for an axis, the circle described has a radius equal to 2 and a diameter equal to 4.

Note here the relation between diameters of first and second circle, namely, $2:1\frac{7}{9}$ equal to 9:8, the very ratio upon which the ratio $3\frac{13}{81}$ is based.

The "artificial root" maintains its superiority over "decimal roots" by the exactness with which the contents of a right-angled triangle is obtained by its use in every case, by the product of the perpendicular and half base; while that of the latter never yields more than an approximation.

It astonished Mr. Theodore Faber no little to find that mathematicians of the calibre of Pascal and Legendre had irrefutably demonstrated the fallacy of the doctrine of a square being divisible into two equal squares, and yet modern science accepts the proposition, "a circumscribed square is equal to twice the inscribed square," as absolute truth.

CHAPTER III

An Explicit Explanation of the First Principles of Mathematics— The Divisibility of the Point and the Line

Any man advancing a pretension like the above is either a sage or a fool! If the former, he will reap a sage's reward, if the latter, a fool's reward. No man of sense and of constitutional diffidence, will expose himself to a fool's reward, and would rather cut off his little finger, than publicly advance mathematical doctrines, the truth of which he could not strictly mathematically demonstrate.

Numerical figures, in themselves, as mere symbolical representatives of quantity, do not possess the attributes of quantity, which are length, breadth and depth: Outside of their application to concrete objects, they have no significance, unless we call a number of thoughts, a quantity. "2 subtracted from 2, is equal to nothing," is an ideal truth, but not a real truth, when applied to objects, since in the realm of objects, there is no such thing as nothing. Subtraction means separation or removal of one object from another; it is another form of division. In division, the quotient does not represent a number of parts of the dividend, but only the *number of times* the divisor is contained in the dividend. We perceive then, that the number expressing the quotient has a very different significance from the number expressing a remainder; the one expresses *times,* and the other *parts.* Annihilation of matter, as such, is not recognized in philosophy. When matter ceases to be matter, it is spirit, for spirit and matter are the only primary elements recognized in philosophy. What is not one, must be the other, and *vice versa.*

We cannot divide 0 or zero, however much mathematicians may attempt such absurdity. Zero is the negation of quantity. To mathematicians, who try to puzzle us with the question, "What is the square root of —a?" we would respectfully suggest that, since a positive must exist in conception before a negative can be predicated of it, they call the *root* positive, but the *square* of the same, negative—thus, $\sqrt{—a} = \sqrt{a—}$; $\sqrt{a—^2} = —a$. Quantity being always a finite conception, and the finite being ever divisible, that is, subject to measure and number, we cannot but see, that harmony between measure and number, that is, between geometry and arithmetic, must be a *law of nature,* since there is nothing indivisible in nature, excluding "space," from our meaning of this word, and the divisible is ever subject to measure and number. We have met with numerous mathematicians, who still stoutly believe in an indivisible line, and no wonder, when the Pythagorean problem itself is based on that doctrine, and is the sole support of the same; yet the great Newton already expressed his *doubts* in the existence of the indivisible in nature as not in harmony with his reason; but his simple *doubt* would today, under the light of our discovery, be quickly converted into *conviction!*

The philosopher Berkeley, conscious of many inconsistencies in the science of mathematics, proposed numerous reforms. We mention these facts to the outside public, that they may understand that the term "positive and exact" is not applicable to the science as a whole, in its present form, and that there is room in it for discoveries! Mathematics is emphatically a science of measures and of numbers, and how the *indivisible line* found a place in it, transcends our comprehension. It is purely ideal conception, not derived from anything *visible,* except as its necessary opposite, hence we scarcely wonder at the idealistic philosophers calling it an innate, inborn idea, not derived through experience! But as the indivisible cannot possibly serve for a basis of a science of measure and of numbers, it is not one of the *first principles* of the science; and space, being absolute, one and indivisible, cannot serve as a first principle of the science, nor can we have any innate, inborn idea of space, since the infant is apt to grasp after the moon for a toy, and therefore has not the slightest notion of *space.*

"Good heavens!" we hear mathematicians exclaim, "what are we coming to? Has not the science all along taught, that pure mathematics deals only with abstract space?" Impossible! We cannot measure space *per se,* but only the things in space and their separation; and the use of the word "space," as a substitute for *surface,* to which alone measure and number are applicable, is highly improper. An object in space is not one quantity, and the space it occupies another quantity! Every measure, as such, is a quantity, and every quantity, as such, may serve as a measure; hence, there is harmony between quantity and measure, and the expression "incommensurable quantity," in itself is a paradox!

Surface, as such, having length and breadth only, is not quantity *per se;* but the cube measure unit is a quantity, and we use one of its surfaces for a measure of surfaces, since like can only be measured with like, and through the medium of the surfaces we measure quantities; hence we call the surface symbolically a quantity. Length, by itself, is not a quantity; a foot long, a yard long, a mile long, expresses only the *side* of a *square* standard measure unit. We image surfaces on paper by *lines,* to represent the boundaries of the surfaces; but a boundary away from its object has only an ideal, or abstract significance, and mathematics, dealing with these ideal surfaces and lines, is therefore called an abstract science. A "line," as representative of a boundary, is not quantitative, but it is quantitative as representative of a *square* *root,* consisting of a straight line of square units, as one line out of a number of equal divisions of a square surface, so that such *line* or *square root,* multiplied by itself, gives the number of square measure units contained in the square, and hence the "square root" may also be called the "side" of the square.

We cannot multiply one side of a geometrical figure with another, without identifying the word "side" with "square root," since multiplication is only possible with *units.* a + b, as representative of a "line," is the inquantitative side of *one* unit, be that a triangle or a square, and as such, cannot be multiplied with a similar side, for 1 × 1 is not *multiplication!* We admire the French division of unit measures into "lines"; but *these* lines *are quantitative,* and have nothing about them calculated to mislead the mind into a paradox, as the "indivisible line" does. We perceive at once, that under this doctrine the side of a square cannot be coincident with the side of a right-angled triangle, while the Pythagorean problem *assumes* such coincidence, *without previous proof,* and bases its whole demonstration upon such coincidence, since it speaks of the *squares* of the *sides* of a right-angled *triangle,* in its very statement. How can the side of a triangle serve as a square root? The law of mathematical demonstration admits of no assumptions without proof, and we hold Pythagoras himself amenable to this law! Prove the coincidence, and we will believe in the truth of the famous problem, although it be equal to saying that twice 2 is 3!

Will mathematicians exclaim, "What becomes, under your doctrine, of the center of the universe—our mathematical point? How can we do without an indivisible point into which all the indivisible radii of the circle center?" What is the primordial form of matter, a cube or a sphere? We would not get far in the construction of a globe by beginning with a cube atom of matter for a center! Hence the center of a globe must necessarily be a spherical atom, and mathematicians will perceive at once that all radii from the periphery of the globe must strike the periphery of the central atom, and afford room for every one of the radii, however much divided; and we may conceive a central atom, within the central atom, to infinity of division, on account of the necessarily infinite divisibility of matter; we say necessarily, because the moment matter ceases to be divisible, it would no longer be matter.

CHAPTER IV

An Explanation of the First Principles of Mathematics—The Origin of the Line

Before closing our remarks on the "first principles of mathematics," we wish to say something on the origin of a *line*. Mathematicians place the origin of a line in the motion of an indivisible point, without length, breadth and depth. How *such* an object can possibly be conceived to have *motion,* transcends altogether our comprehension. If we were to define *nothing,* although that word has in reality no predicate, we should just use the definition of a "mathematical point," and what philosopher would ascribe motion to *nothing!* What a *basis* for a science of measures and of numbers! The legitimate basis of that science is a square unit, infinitely divisible, so that a line of infinitesimal square units, might even become invisible, without losing the characteristic of entity, and such a line should satisfy the idealist. But a square unit, being infinitely divisible in its angles, and, hence, not closing in an indivisible point, is *not absolute,* but is *all but absolute.* We approach the absolute by division, and recede from it by multiplication. We shall refer to this fact again, in our analysis of the hypothenuse, together with its square, so called.

We perceive that the divisible is a *separation* from the indivisible, but not so, as to destroy all relation; in fact, the divisible could not exist, if the indivisible did not first exist; hence the two have the relation of cause and effect. Then might we not say, that a line is the effect of an indivisible point? But the indivisible transcends all our powers of comprehension, and a *line* is a thing that we have to comprehend *as it is,* or we know nothing of it, and can do nothing with it, in a science of measures and of numbers, which deals only with the finite and limited—the divisible. Hence we perceive that an indivisible line, becomes by pure reflection upon itself, an impracticable object, and we shall further on, mathematically prove its impossibility.

If the square unit cannot possibly be absolute in its angles, then the *side* of a square unit, or of a square consisting of square units, cannot be an absolute unity, though it may *appear* so to our vision; nor can a plane surface be absolute. In fact, under an adequate magnifier, the most exquisitely polished surface of the densest material would appear waving in its character. How is this to be accounted for? It can only be accounted for by the fact of the primary *spherical form* of material atoms. These are so minute, that a plane surface composed of them appears to our limited power of vision as a perfectly continuous, absolute plane. And such *appearance* of surface we *measure,* and we measure it with the *apparent* surface of the measure unit, since like can only be measured with like. We might perhaps get as far in geometry, by the use of a circular unit measure, provided it be infinitesimal enough to obliterate to our vision the distinction between a curve and a straight line! What is apparently the longest and straightest line in nature? Is it not the sunbeam? Yet Sir Isaac Newton calls it a wave line! And probably with perfect justice, since, if light be material, it must be composed of spherical atoms, and a sunbeam may thus be looked upon as an immensely long string of bead-like atoms.

We cannot in this essay dwell on all the philosophical deductions to be drawn from our reflections; we will, however, mention that spherical atoms serve as an adequate explanation of the *porosity* of the *densest* masses of matter. The philosopher Fichte placed the curious question before mathematicians, "Are there any other than straight lines?" Our above remarks are a full answer to that question! We might, Yankee-like, retort with the question, "Are there any other but curve lines?" We use the water level for a standard of level, while we *know* that every water-line must be a *curve line.* What is the origin of a line? Man's knowledge is almost exclusively limited to things as they appear, and not as they are in themselves. This is never to be ignored in science. We perceive that our perception of straightness is totally dependent on our sense of vision, and hence we can have no innate, inborn idea of a straight line! The idealistic philosophers will have to "cave in"! And we rather regret the necessity, since we would feign have "fought on the side of our fathers." But truth knows no fatherland. The sun of truth shines for the whole universe!

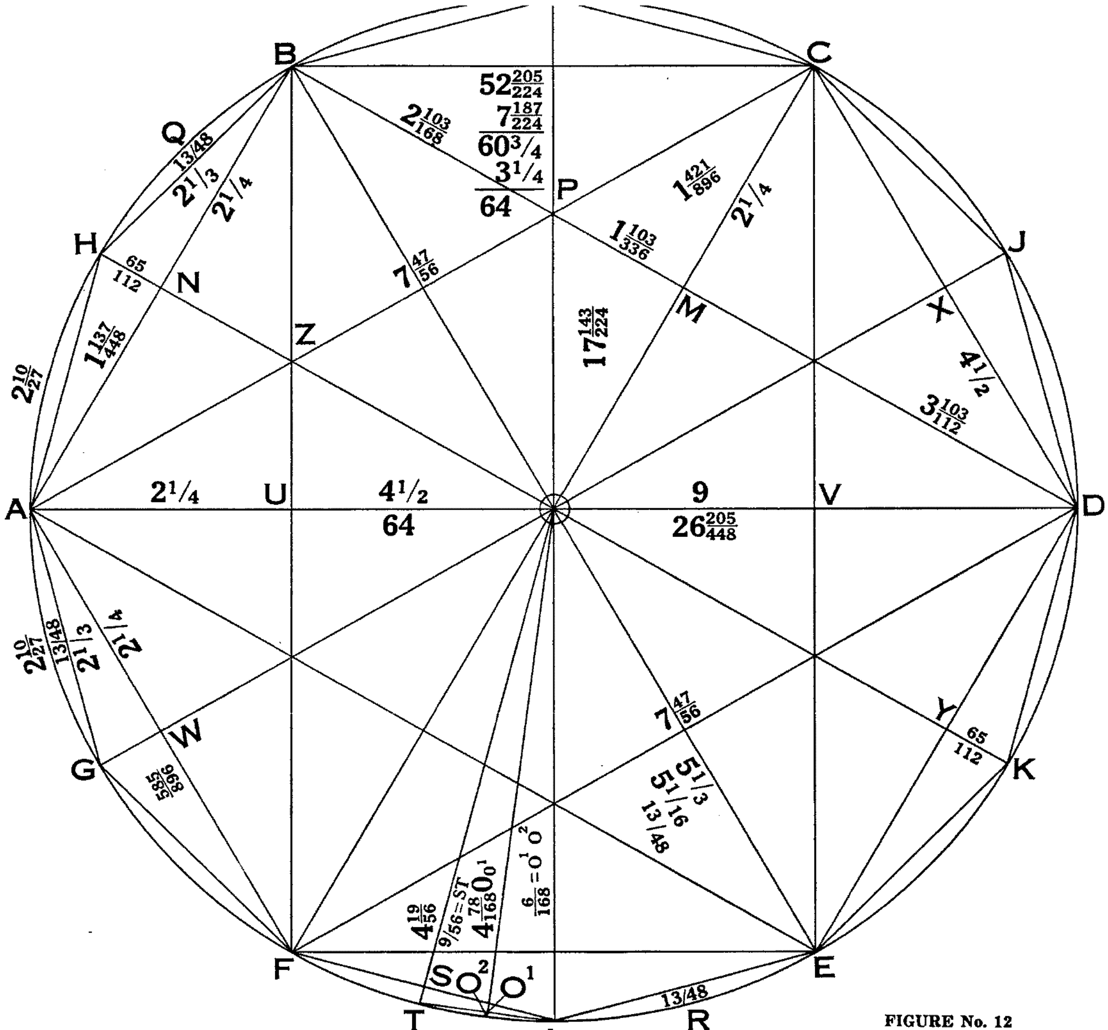
SECOND SECTION

Exact Diagrams and Numerical Demonstrations

By

Carl Theodore Heisel

Proving Beyond Refutation
The Exact Ratio of Diameter to Circumference
and the
Square of Equal Area of Any Circle
with Many New and Original Theorems and Propositions
Hitherto Unknown to the Scientific World, and
Disproving the
World Renowned Pythagorean Problem
Accepted as Absolute by Geometricians for the
Last Twenty-Four Centuries.


A
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X
Y
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O¹
O²
$2\frac{10}{27}$
$2\frac{10}{27}$
13/48
13/48
13/48
$2^1/3$
$2^1/3$
$2^1/4$
$2^1/4$
$2^1/4$
$2^1/4$
$1\frac{137}{448}$
$\frac{65}{112}$
$\frac{65}{112}$
$\frac{585}{896}$
$52\frac{205}{224}$
$7\frac{187}{224}$
$60^3/4$
$3^1/4$
64
$2\frac{103}{168}$
$7\frac{47}{56}$
$7\frac{47}{56}$
$1\frac{421}{896}$
$1\frac{103}{336}$
$17\frac{143}{224}$
$4^1/2$
$4^1/2$
64
9
$26\frac{205}{448}$
$3\frac{103}{112}$
$5^1/3$
$5^1/16$
13/48
$4\frac{19}{56}$
9/56=ST
$4\frac{78}{168}O_0{}^1$
$\frac{6}{168}=O^1\ O^2$

FIGURE No. 12

Length of Lines in Figure No. 12

Exact Length of Lines in Figure No. 12 with Regular Methods or Rules for Obtaining the Same

9 = Diameter.

$4\frac{1}{2}$ = Radius $= 9 \div 2 = 4\frac{1}{2}$.

$2\frac{1}{4}$ = Half Radius $= 4\frac{1}{2} \div 2 = 2\frac{1}{4}$.

$28\frac{4}{9}$ = Circumference of Circle of Diameter of $9 = 9 \times 3\frac{13}{81} = 28\frac{4}{9}$.

28 = Perimeter of Dodecagon = HB $\times 12 = 2\frac{1}{3} \times 12 = 28$.

27 = Perimeter of Hexagon = diameter $\times 3 = 9 \times 3 = 27$.

$7\frac{1}{9}$ = AI, etc., = Arc of Quadrant = $\frac{1}{4}$ of Circumference $= 28\frac{4}{9} \div 4 = 7\frac{1}{9}$.

$7\frac{1}{9}$ = AI, etc., = Arc of Quadrant = Arc AH $\times 3 = 2\frac{10}{27} \times 3 = 7\frac{1}{9}$.

$7\frac{1}{9}$ = AI, etc., = Arc of Quadrant = Radius multiplied by Pi multiplied by 90° and divided by 180° $= 4\frac{1}{2} \times 3\frac{13}{81} \times 90^\circ \div 180^\circ = 7\frac{1}{9}$.

$7\frac{47}{56}$ = AC, etc., = Side of Right Angle Triangle ACD = square root of diameter square minus CD square $= \sqrt[2]{9^2 - 4\frac{1}{2}^2} = 81 - 20\frac{1}{4} = 60\frac{3}{4} = \sqrt[2]{60\frac{3}{4}} = 7\frac{47}{56}$.

$7\frac{47}{56}$ = AC, etc., = side of inscribed triangle = square root of $\frac{3}{4}$ of square of diameter $= 9 \times 9 = 81 \times \frac{3}{4} = 81 - 20\frac{1}{4} = 60\frac{3}{4} = \sqrt[2]{60\frac{3}{4}} = 7\frac{47}{56}$.

$6\frac{3}{4}$ = AV, etc., = $\frac{3}{4}$ of diameter $= 2\frac{1}{4} \times 3 = 6\frac{3}{4}$.

$3\frac{103}{112}$ = MD, etc., $= \sqrt[2]{CD^2 - CM^2} = \sqrt[2]{4\frac{1}{2}^2 - 2\frac{1}{4}^2} = 3\frac{103}{112}$.

$3\frac{103}{112}$ = MD, etc., = $\frac{1}{2}$ of BD $= 7\frac{47}{56} \div 2 = 3\frac{103}{112}$.

$3\frac{103}{112}$ = MD, etc., $= AD : BD :: OD : MD = 9 : 7\frac{47}{56} :: 4\frac{1}{2} : 3\frac{103}{112}$.

$2\frac{103}{168}$ = BP, etc., = $\frac{1}{3}$ BD $= 7\frac{47}{56} \div 3 = 2\frac{103}{168}$.

$2\frac{103}{168}$ = BP, etc., = BP = PC. PC $= \sqrt[2]{PM^2 + CM^2} = \sqrt[2]{1\frac{103}{336}^2 + 2\frac{1}{4}^2} = 2\frac{103}{168}$.

$1\frac{103}{336}$ = PM, etc., = $\frac{1}{6}$ of BD $= 7\frac{47}{56} \div 6 = 1\frac{103}{336}$.

$1\frac{103}{336}$ = PM, etc., $= \sqrt[2]{PC^2 - MC^2} = \sqrt[2]{2\frac{103}{168}^2 - 2\frac{1}{4}^2} = 1\frac{103}{336}$.

$2\frac{10}{27}$ = HQB, etc., = $\frac{1}{12}$ of circumference $= 28\frac{4}{9} \div 12 = 2\frac{10}{27}$.

$2\frac{10}{27}$ = HQB, etc., = Radius multiplied by Pi multiplied by 30° divided by 180° $= 4\frac{1}{2} \times 3\frac{13}{81} \times 30^\circ \div 180^\circ = 2\frac{10}{27}$, $= 4\frac{1}{2} \times 3\frac{13}{81} = \frac{1152}{81} \times 30^\circ = \frac{34560}{81} \div 180^\circ = \frac{192}{81} = 2\frac{30}{81} = 2\frac{10}{27}$.

$2\frac{1}{4}$ = NB, etc., = $\frac{1}{2}$ Radius $= 4\frac{1}{2} \div 2 = 2\frac{1}{4}$.

$2\frac{1}{4}$ = AU, etc., = $\frac{1}{2}$ Radius $= 4\frac{1}{2} \div 2 = 2\frac{1}{4}$.

$2\frac{1}{3}$ = HB, etc., $= \sqrt[2]{HN^2 + NB^2} = \sqrt[2]{\frac{65}{112}^2 + 2\frac{1}{4}^2} = 2\frac{1}{3}$.

$2\frac{1}{3}$ = HB, etc., = $\frac{1}{12}$ Perimeter of Dodecagon $= \frac{1}{12}$ of $28 = 2\frac{1}{3}$.

$2\frac{1}{3}$ = HB, etc., = $\frac{1}{12}$ difference in length of Perimeter of Dodecagon and Hexagon added to $\frac{1}{2}$ radius = 28 — 27 = 1. $1 \div 12 = \frac{1}{12} + \frac{1}{2}$ of radius $= \frac{1}{12} + 2\frac{1}{4} = 2\frac{1}{3}$.

$\frac{65}{112}$ = HN, etc., = HO — NO $= 4\frac{1}{2} - 3\frac{103}{112} = \frac{65}{112}$.

$\frac{65}{112}$ = HN, etc., $= \sqrt[2]{HB^2 - NB^2} = \sqrt[2]{2\frac{1}{3}^2 - 2\frac{1}{4}^2} = \frac{65}{112}$.

$4\frac{19}{56}$ = OS, etc., = height of triangle OFL $= \sqrt[2]{OF^2 - FS^2} = \sqrt[2]{4\frac{1}{2}^2 - 1\frac{1}{6}^2} = 4\frac{19}{56}$.

$\frac{9}{56}$ = ST, etc., = height of triangle FTL, etc., = OT — OS $= 4\frac{1}{2} - 4\frac{19}{56} = \frac{9}{56}$.

$4\frac{78}{168}$ = Oo^1, etc., = height of 24 sided polygon.

$\frac{6}{168}$ = o^1o^2, etc., = height of triangle $To^2L = 4\frac{1}{2} - 4\frac{78}{168} = \frac{6}{168}$.

Exact Area of Triangles in Figure No. 12

With Regular Methods and Rules for Obtaining the Same

$26\frac{205}{448}$ = ACE, etc., = CE × ½ AV $= 7\frac{47}{56} \times 3\frac{3}{8}$ = base by ½ height $= 26\frac{205}{448}$.

$26\frac{205}{448}$ = ACE, etc., = CPM × 18 $= 1\frac{421}{896} \times 18 = 26\frac{205}{448}$.

$17\frac{143}{224}$ = ACD, etc., = CPM × 12 $= 1\frac{421}{896} \times 12 = 17\frac{143}{224}$.

$17\frac{143}{224}$ = ACD, etc., = AD × ½ CV $= 9 \times 1\frac{215}{224}$ = base by ½ height $= 17\frac{143}{224}$.

$17\frac{143}{224}$ = ACD, etc., = AC × ½ CD $= 7\frac{47}{56} \times 2\frac{1}{4}$ = base by ½ height $= 17\frac{143}{224}$.

$8\frac{367}{448}$ = BCO, etc., = CPM × 6 $= 1\frac{421}{896} \times 6 = 8\frac{367}{448}$.

$8\frac{367}{448}$ = BCO, etc., = ½ CO × BM $= 2\frac{1}{4} \times 3\frac{103}{112}$ = ½ base by height $= 8\frac{367}{448}$.

$4\frac{367}{896}$ = ANO, etc., = ½ AN × NO $= 1\frac{1}{8} \times 3\frac{103}{112}$ = base by ½ height $= 4\frac{367}{896}$.

$1\frac{421}{896}$ = CPM, etc., $= 1\frac{103}{336} \times 1\frac{1}{8}$ = base × ½ height $= 1\frac{421}{896}$.

$4\frac{367}{896}$ = ANO, etc., = ANZ × 3 $= 1\frac{421}{896} \times 3 = 4\frac{367}{896}$.

$1\frac{137}{448}$ = AHB, etc., $= \frac{65}{112} \times 2\frac{1}{4}$ = height by ½ base $= 1\frac{137}{448}$.

$1\frac{137}{448}$ = AHB, etc., = AHN × 2 $= \frac{585}{896} \times 2 = \frac{1170}{896} = 1\frac{137}{448}$.

$\frac{585}{896}$ = FGW, etc., = ½ FW × GW $= 1\frac{1}{8} \times \frac{65}{112}$ = ½ base by height $= \frac{585}{896}$.

$\frac{585}{896}$ = FGW, etc., = AHB ÷ 2 $= 1\frac{137}{448} \div 2 = \frac{585}{896}$.

$5\frac{1}{16}$ = HBO, etc., = ½ HB × OS $= 1\frac{1}{6} \times 4\frac{19}{56}$ = ½ base by height $= \frac{486}{112} \times \frac{7}{6} = \frac{3402}{672} = 5\frac{1}{16}$.

$5\frac{1}{16}$ = HBO, etc., = NBO + HNB $= 4\frac{367}{896} + \frac{585}{896} = 5\frac{1}{16}$.

$5\frac{1}{16}$ = HBO, etc., = ½ HO × NB $= 2\frac{1}{4} \times 2\frac{1}{4} = 5\frac{1}{16}$.

$5\frac{1}{16}$ = HBO, etc., = ¼ radius square $= 4\frac{1}{2}^2 = 20\frac{1}{4} \div 4 = 5\frac{1}{16}$.

64 = area of circle of diameter of 9 $= 28\frac{4}{9} \times 2\frac{1}{4} = 64$.

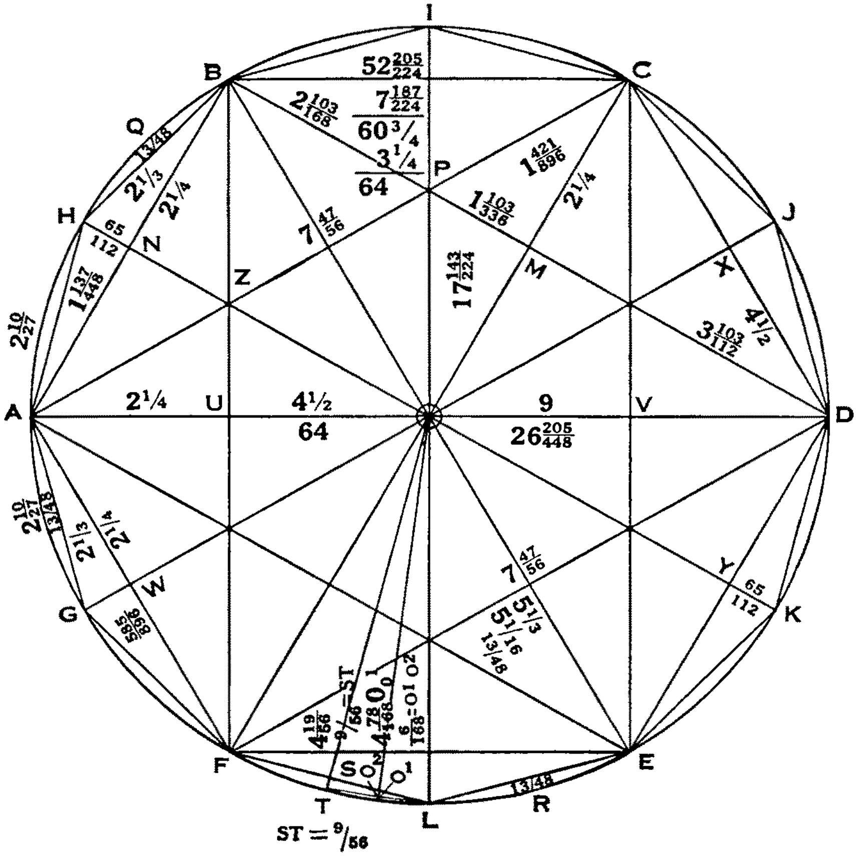

Figure No. 12

CHAPTER IX

Rules for Squaring the Circle Numerically

Length of Lines and Areas of Triangles, Sectors, Segments, Quadrants, Etc., in Figure 12, with Regular Methods and Rules for Obtaining the Same

1. The whole is equal to the sum of all of its parts.
2. Things equal to the same thing are equal to each other.
3. Any law true of one circle is true of all other circles.

$3\frac{13}{81}$ = ratio of diameter of any circle to circumference of same.

$9 : 8$ = ratio of diameter of any circle to side of square of equal area.

$8 = \frac{8}{9}$ of 9 = exact side of square of equal area of circle of diameter of 9.

64 = area of circle of diameter of $9 = 8 \times 8 = 64$.

64 = area of circle of diameter of 9 = circumference $\times$ $\frac{1}{2}$ radius $= 28\frac{4}{9} \times 2\frac{1}{4} = 64$.

$28\frac{4}{9}$ = circumference of circle of diameter of $9 = 3\frac{13}{81} \times 9 = 28\frac{4}{9}$.

16 = area of quadrant of circle of diameter of 9 = arc of quadrant $\times$ $\frac{1}{2}$ radius $= 7\frac{1}{9} \times 2\frac{1}{4} = 16$.

$7\frac{1}{9}$ = arc of quadrant of circle of diameter of $9 = 28\frac{4}{9} \div 4 = 7\frac{1}{9}$.

$2\frac{10}{27}$ = arc of $30°$ = $\frac{1}{12}$ of circumference $= 28\frac{4}{9} \div 12 = 2\frac{10}{27}$.

$\frac{13}{48}$ = area of segment HQB = radius square multiplied by $\frac{13}{81} \div 12$ $= 4\frac{1}{2}^2 = 20\frac{1}{4} \times \frac{13}{81} = 3\frac{1}{4} \div 12 = \frac{13}{48}$.

$\frac{13}{48}$ = area of segment HQB = sector OHQB minus triangle OHB $= 5\frac{1}{3} - 5\frac{1}{16} = \frac{13}{48}$.

$5\frac{1}{3}$ = area of sector OHQB $= 30°$ sector of circle of diameter of 9 = arc $\times$ $\frac{1}{2}$ radius $= 2\frac{10}{27} \times 2\frac{1}{4} = 5\frac{1}{3}$.

$5\frac{1}{3}$ = area of sector OHQB $= \frac{1}{12}$ of area of circle $\frac{1}{12}$ of $64 = 5\frac{1}{3}$.

$5\frac{1}{3}$ = area of sector OHQB $= 360° : 30° :: 64 : 5\frac{1}{3}$.

$5\frac{1}{16}$ = area of triangle OHB $= \frac{1}{2}$ base $\times$ height $= 2\frac{1}{4} \times 2\frac{1}{4} = 5\frac{1}{16}$.

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1. The area of any circle is equal to its circumference multiplied by ½ of its radius.

Diameter		Circumference		Half Radius	Area	Diameter		Circumference		Half Radius	Area
1	=	$3\frac{13}{81}$	×	$\frac{1}{4}$ =	$\frac{64}{81}$	6	=	$18\frac{78}{81}$	×	$1\frac{1}{2}$ =	$28\frac{36}{81}$
2	=	$6\frac{26}{81}$	×	$\frac{1}{2}$ =	$3\frac{13}{81}$	7	=	$22\frac{10}{81}$	×	$1\frac{3}{4}$ =	$38\frac{58}{81}$
3	=	$9\frac{39}{81}$	×	$\frac{3}{4}$ =	$7\frac{9}{81}$	8	=	$25\frac{23}{81}$	×	2 =	$50\frac{46}{81}$
4	=	$12\frac{52}{81}$	×	1 =	$12\frac{52}{81}$	9	=	$28\frac{36}{81}$	×	$2\frac{1}{4}$ =	64
5	=	$15\frac{65}{81}$	×	$1\frac{1}{4}$ =	$19\frac{61}{81}$	10	=	$31\frac{49}{81}$	×	$2\frac{1}{2}$ =	$79\frac{1}{81}$

2. The diameter of any circle is to its circumference, as the square of its radius is to its area.

Diameter		Circumference		Radius Square		Area	Diameter		Circumference		Radius Square		Area
1	:	$3\frac{13}{81}$	::	$\frac{1}{4}$	:	$\frac{64}{81}$	6	:	$18\frac{78}{81}$	::	9	:	$28\frac{36}{81}$
2	:	$6\frac{26}{81}$	::	1	:	$3\frac{13}{81}$	7	:	$22\frac{10}{81}$	::	$12\frac{1}{4}$	:	$38\frac{58}{81}$
3	:	$9\frac{39}{81}$	::	$2\frac{1}{4}$	:	$7\frac{9}{81}$	8	:	$25\frac{23}{81}$	::	16	:	$50\frac{46}{81}$
4	:	$12\frac{52}{81}$	::	4	:	$12\frac{52}{81}$	9	:	$28\frac{36}{81}$	::	$20\frac{1}{4}$	:	64
5	:	$15\frac{65}{81}$	::	$6\frac{1}{4}$	:	$19\frac{61}{81}$	10	:	$31\frac{49}{81}$	::	25	:	$79\frac{1}{81}$

3. The area of any circle is equal to the arc of its quadrant multiplied by its diameter.

Arc of Quadrant		Diameter		Area	Arc of Quadrant		Diameter		Area
$\frac{64}{81}$	×	1	=	$\frac{64}{81}$	$4\frac{60}{81}$	×	6	=	$28\frac{36}{81}$
$1\frac{47}{81}$	×	2	=	$3\frac{13}{81}$	$5\frac{43}{81}$	×	7	=	$38\frac{58}{81}$
$2\frac{30}{81}$	×	3	=	$7\frac{9}{81}$	$6\frac{26}{81}$	×	8	=	$50\frac{46}{81}$
$3\frac{13}{81}$	×	4	=	$12\frac{52}{81}$	$7\frac{9}{81}$	×	9	=	64
$3\frac{77}{81}$	×	5	=	$19\frac{61}{81}$	$7\frac{73}{81}$	×	10	=	$79\frac{1}{81}$

4. The square of the diameter of any circle multiplied by the area of circle of diameter of 1, equal to $\frac{64}{81}$, equals the area of any such circle.

Diameter	Square of Diameter		Area of Diameter of One		Area of Circle		Area of Circl
2	$2 \times 2 = 4$	$\times$	$\frac{64}{81}$	=	$\frac{256}{81}$	=	$3\frac{13}{81}$
3	$3 \times 3 = 9$	$\times$	$\frac{64}{81}$	=	$\frac{576}{81}$	=	$7\frac{9}{81}$
4	$4 \times 4 = 16$	$\times$	$\frac{64}{81}$	=	$\frac{1024}{81}$	=	$12\frac{52}{81}$
5	$5 \times 5 = 25$	$\times$	$\frac{64}{81}$	=	$\frac{1600}{81}$	=	$19\frac{61}{81}$
6	$6 \times 6 = 36$	$\times$	$\frac{64}{81}$	=	$\frac{2304}{81}$	=	$28\frac{36}{81}$
7	$7 \times 7 = 49$	$\times$	$\frac{64}{81}$	=	$\frac{3136}{81}$	=	$38\frac{58}{81}$
8	$8 \times 8 = 64$	$\times$	$\frac{64}{81}$	=	$\frac{4096}{81}$	=	$50\frac{46}{81}$
9	$9 \times 9 = 81$	$\times$	$\frac{64}{81}$	=	$\frac{5184}{81}$	=	64
10	$10 \times 10 = 100$	$\times$	$\frac{64}{81}$	=	$\frac{6400}{81}$	=	$79\frac{1}{81}$

5. The area of any circle is equal to $\frac{1}{4}$ of its circumference multiplie by its diameter.

One-Fourth of Circumference		Diameter	Circle Area	Circle Area	One-Fourth of Circumference		Diameter	Circle Area	Circle Area
$\frac{64}{81}$	$\times$	1 =	$\frac{64}{81}$ =	$\frac{64}{81}$	$\frac{384}{81}$	$\times$	6 =	$\frac{2304}{81}$ =	$28\frac{36}{8}$
$\frac{128}{81}$	$\times$	2 =	$\frac{256}{81}$ =	$3\frac{13}{81}$	$\frac{448}{81}$	$\times$	7 =	$\frac{3136}{81}$ =	$38\frac{58}{8}$
$\frac{192}{81}$	$\times$	3 =	$\frac{576}{81}$ =	$7\frac{9}{81}$	$\frac{512}{81}$	$\times$	8 =	$\frac{4096}{81}$ =	$50\frac{46}{8}$
$\frac{256}{81}$	$\times$	4 =	$\frac{1024}{81}$ =	$12\frac{52}{81}$	$\frac{576}{81}$	$\times$	9 =	$\frac{5184}{81}$ =	64
$\frac{320}{81}$	$\times$	5 =	$\frac{1600}{81}$ =	$19\frac{61}{81}$	$\frac{640}{81}$	$\times$	10 =	$\frac{6400}{81}$ =	$79\frac{1}{81}$

6. The area of any circle is always a^2 of the formula $a^2 + 2ab + b^2$ o the square of the diameter of such circle, viz.: The square of th diameter of a circle of diameter of 9, $= 9 \times 9 = 81$. $a = \frac{8}{9}$ of $= 8$. $a^2 = 8 \times 8 = 64$. $2ab =$ twice $1 \times 8 = 16$. $b^2 = 1 =$ $64 + 16 + 1 = 81$.

7. $\frac{64}{81}$ = area of a circle of diameter of 1, consequently $\frac{64}{81}$ of th square of the diameter of any circle equals the area of such circl viz.: Square of diameter of 9 = 81. $\frac{64}{81}$ of 81 = 64, equals are of circle of diameter of 9.

8. The square of the diameter of any circle, minus $\frac{17}{81}$ of such squar equals the exact area of such circle, viz.: The square of diameter c 100 = 10,000. $\frac{17}{81}$ of 10,000 = $2098\frac{62}{81}$.

$$\left.\begin{array}{l}\frac{17}{81} \text{ of } 10{,}000 = 2098\frac{62}{81} \\ \frac{64}{81} \text{ of } 10{,}000 = \underline{7901\frac{19}{81}} \\ \qquad\qquad\quad\;\; 10{,}000\end{array}\right\} \frac{17}{81} + \frac{64}{81} = \frac{81}{81}.$$

$\frac{8}{9}$ of 100 = $88\frac{8}{9}$. $88\frac{8}{9}^2 = 7901\frac{19}{81}$ = exact area of circle of dian eter of 100.

9. The area of a 30° sector of any circle multiplied by 12, equals are of such circle, viz.: Area of 30° sector of circle of diameter of 9 = area OHQB Figure 12 = $5\frac{1}{3} \times 12 = 64$ = area of circle of dian eter of 9.

10. Mathematicians who prefer to use the decimal fraction, with its approximate or infinite results, in preference to exact results, may do so to their heart's content, by converting the exact fraction $^{64}/_{81}$ into a decimal equal to $.79+$ or $^{79+}/_{100}$, which is an irrational number lacking one unit of measure from being a square, and is a repeating decimal, and still get results within $^{1}/_{10000}$ of a true result. But why not always be exact in a Positive and Exact Science, viz.:

$81)64.0(.7901234567901\,^{19}/_{81}$ = a repeating decimal.

```
81)64.0(.7901234567901 19/81
   567
    730
    729
     100
      81
      190
      162
       280
       243
        370
        324
         460
         405
          550
          486
           640
           567
            730
            729
             100
              81
             19/81
```

$^{64}/_{81}$ and $^{79+}/_{100} = {}^{6400}/_{8100} - {}^{6399}/_{8100}$ = difference $= {}^{1}/_{8100}$

```
  64        79+       81
 100        81       100
6400        79+     8100
           632
          6399+
```

Decimal reduced to fraction:

```
.7901 19/81        1.0000
   81                 81
   19             81.0000
 7901
63208

64.0000
------- = 64/81
81.0000
```

11. Proving that the area of a circle of diameter of 9 inches equals exactly 64 inches. The circumference of a circle of 9 inches equals $28\frac{4}{9}$ inches, equal to $^{256}/_{9}$. $^{1}/_{64}$ of the circumference equals an arc $^{4}/_{9}$ of an inch long, $= {}^{256}/_{9} \div 64 = {}^{4}/_{9}$. The sector with an arc $^{4}/_{9}$ inches long multiplied by $\frac{1}{2}$ radius equals the area of such sector, viz.: $^{4}/_{9} \times 2\frac{1}{4} = {}^{4}/_{9} \times {}^{9}/_{4} = {}^{36}/_{36} = 1$ square inch, and 1 square inch multiplied by $64 = 64$ square inches, and is the exact area of a circle of diameter of 9.

12. The whole is equal to the sum of all its parts.

The area of any circle equals the sum of the area of all its parts.

The area of a circle of diameter of 9 equals 64, viz.:

The area of the inscribed hexagon ABCDEF equals.... $52\frac{205}{224}$ and is composed of 36 triangles similar to CPM, the area of which is $1\frac{421}{896}$, and the area $1\frac{421}{896}$ multiplied by 36 equals $52\frac{205}{224}$ (see Figure No. 12).

The area of the inscribed dodecagon (Figure No. 12) equals the area of the inscribed hexagon plus the area of 6 triangles similar to AHB, each with an area of $1\frac{137}{448}$, and the area $1\frac{137}{448}$ multiplied by 6 equals $7\frac{187}{224}$, added to the area of hexagon equals $60\frac{3}{4}$, equal to area of inscribed dodecagon. $7\frac{187}{224}$

———

The area of the circle of diameter of 9 equals the area of its inscribed dodecagon plus the area of 12 segments similar to HQB, with an area of $\frac{13}{48}$ each, and $\frac{13}{48}$ multiplied by 12 equals an area of $3\frac{1}{4}$, added to the area of inscribed dodecagon equals area of circle of diameter of 9. $60\frac{3}{4}$ $3\frac{1}{4}$

———

64