THE ELEMENTS OF DYNAMIC SYMMETRY

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INTRODUCTION

Synthesis and Analysis.—The Difference between Static and Dynamic Symmetry.—Sources for the Study of Dynamic Symmetry.

THE basic principles underlying the greatest art so far produced in the world may be found in the proportions of the human figure and in the growing plant. These principles have been reduced to working use and are being employed by a large number of leading artists, designers, and teachers of design and manual art.

The principles of design to be found in the architecture of man and of plants have been given the name "Dynamic Symmetry." This symmetry is identical with that used by Greek masters in almost all the art produced during the great classical period.

The synthetic use of these design principles is simple. The Greeks probably used a string held in the two hands. The Harpedonaptae or rope-stretchers of Egypt had no other instrument for orientating and surveying or laying out temple plans. The recovery of these design principles by analysis is difficult, requiring special talent and training, considerable mathematical ability, much patience and sound aesthetic judgment. The analysis of the plan of a large building, such for example as the Parthenon, often is not so difficult as the recovery of the plans of many minor design forms. Sometimes a simple vase is most baffling, requiring days of intensive inspection before the design theme becomes manifest.

To recover these themes of classic design it is necessary to use arithmetical analysis. Geometrical analysis is misleading and inexact. Necessity compelled the old artists to use simple and understandable shapes to correlate the elements of their design fabrics. It is evident that even the simplest pattern arrangements can become very complicated as a design develops. A few lines, simple as a synthetic evolution, may tax the utmost ingenuity to analyze.

The determination of the form principles in a specific example of design means, in a sense, the elimination of the personal element.

With this element removed the residue represents merely the planning knowledge possessed by the artist. This residue is sometimes meagre and more or less meaningless; often it is rich in suggestion and positive design knowledge. Invariably the higher or more perfect the art, the richer is the remainder when the personal element is removed. Also the degree of planning knowledge is positive evidence of the conscious or unconscious use of a scheme in a work of art.

Saracenic, Mahomedan, Chinese, Japanese, Persian, Hindu, Assyrian, Coptic, Byzantine, and Gothic art analyses show unmistakably the conscious use of plan schemes and all belong to the same type. Greek and Egyptian art analyses show an unmistakable conscious use of plan schemes of another type. There is no question as to the relative merit of the two types. The latter is immeasurably superior to the former. This is made manifest as soon as the two types are tested by nature.

These plan schemes, which we find so abundantly in art, are nothing more than symmetry, using the word in the Greek sense of analogy; literally it signifies the relationship which the composing elements of form in design, or in an organism in nature, bear to the whole. In design it is the thing which governs the just balance of variety in unity.

The investigation of this impersonal aspect of art in relation to the symmetry of natural form was begun some twenty years ago. The results of the labor showed clearly that there were but two types of symmetry in nature which could be utilized in design. One of these types, because of its character, was termed "static," the other "dynamic." Possibly the former is but a special case of the latter, as a circle for example, is a special case of an ellipse. At any rate there is no question of the superiority of the dynamic over the static.

The static is the type which can be used both consciously and unconsciously in art. In fact no design is possible without symmetry. The savage decorating his canoe or paddle, his pottery or his blanket, uses static symmetry unconsciously. The crude drawings of the caveman disclose no design, consequently no symmetry.

As civilization advances the artist becomes more or less conscious of the necessity for symmetry or that quality in a work of art or craft which we recognize as design.

When we reach a period which is recognizable as an art epoch, where a people's character is shot through and through a great design fabric and the result stands as a national aesthetic expression, we find invariably, a highly sophisticated use of symmetry. But still it is almost always static.

Static symmetry, as the name implies, is a symmetry which has a sort of fixed entity or state. It is the orderly arrangement of units of form about a center or plane as in the crystal. A snow crystal furnishes a perfect example. It is apparent in cross sections of certain fruits. Diatoms and radiolaria supply other examples. It is the spontaneous type; *i.e.*, an artist or craftsman may use it unconsciously.

Static symmetry, as used by the Copts, Byzantines, Saracens, Mahomedans and the Gothic and Renaissance designers, was based upon the pattern properties of the regular two-dimensional figures such as the square and the equilateral triangle. The static symmetry used by the Greeks, before they obtained knowledge of dynamic symmetry, depended upon an area being divided into even multiple parts, such as a square and a half, three-quarters, one-quarter, one-third, two-thirds, etc.

The Tenea Apollo, a 6th century B. C. Greek statue, has a symmetry theme of three and two-thirds. This means that if a rectangle is made, the area of which consists of three squares and two-thirds of a square, it will be exactly the shape which will contain the projection of the statue. When we find that every member of the body is expressible in terms of this shape and that the theme produces a simple pattern form throughout we decide that this Apollo belongs to the static class.

It is well understood that the archaic Apollo statues of Greece closely followed an Egyptian prototype. A statue of Amenophis IV, of the 14th or 15th century B. C., which is such a prototype, discloses quite unmistakably a dynamic theme.

Diodorus Siculus, the Sicilian Greek historian, says that the early

Greeks obtained their sculptural knowledge from Egypt and tells the story of a certain Rhoecus, a great sculptor who learned his art in Egypt. He had two sons, Telecles and Theodorus, who like their father were sculptors. One of these sons worked at Samos, the other at Ephesus and between them they made a statue according to a prearranged plan. When the two parts of the figure were brought together they fitted exactly, so that the statue appeared the work of one man. Stories of this character suggest that the poet was possibly near the truth when he said:

"From Egypt arts their progress made to Greece,
Wrapt in the fable of the Golden Fleece."

Now that dynamic symmetry supplies a means by which we may closely inspect any design and classify it according to its symmetry theme, we see that the Tenea Apollo, though closely resembling the outward aspect of an Egyptian prototype, such as the statue of Amenophis IV, is in reality, as far as its symmetry is concerned, quite a different thing. The archaic Apollo sculptors of Greece apparently did not have knowledge of the Egyptian symmetry secret when the Tenea figure was made. When we find that the identical symmetry theme disclosed by the Tenea statue was used by Greeks to a limited extent, less than five per cent throughout the classical period, we conclude that this represents the work of a small number of designers who, apparently, did not know the dynamic scheme. Probably they did not belong to the craft guilds. After the decline of Athens and during the Hellenistic Age, we find this Tenea figure or static symmetry type used more and more until by the 1st century B. C. it is the only type observable; the dynamic type in all design entirely disappears at this date.

Curiously, there were but two peoples who did use dynamic symmetry, the Egyptians and the Greeks. It was developed by the former very early as an empiric or rule-of-thumb method of surveying. Possibly the date is as early as the first or second dynasty. Later it was taken over as a means of plan-making in architecture and design in general. The Egyptians seemed to attach some sort of ritualistic significance to the idea as it is found employed in this sense in temple and tomb, particularly in the bas-reliefs which were used so plentifully to adorn these. It is also curious that the Hindus, about the 5th or 8th century B. C., possessed a slight knowledge of dynamic symmetry. A few of the dynamic shapes were actually worked out and appear in the Sulvasutras, literally "the rules of the cord," and were part of a sacrificial altar ritual. But to what extent it may have been used in Hindu art is not known, because examples containing its presence have disappeared.

The Greeks obtained knowledge of dynamic symmetry from the Egyptians some time during the 6th century B. C. It supplanted, probably rapidly, a sophisticated type of static symmetry then in general use. In Greece, as in India and in Egypt, the scheme was connected with altar ritual. Witness the Delian or Duplication of the Cube problem. The Greeks, however, soon far outstripped their Egyptian masters and within a few years after acquiring the knowledge, apparently made the astounding discovery that this symmetry was the symmetry of growth in man.

According to Vitruvius the Greeks learned symmetry from the human figure and were most particular in applying it to their works of art, especially to their temples. This, however, is not more reliable than other Vitruvian statements. The Roman architect had no knowledge of symmetry beyond a crude form of the static. He declared that the Greeks used a module to determine the symmetry of their temples and gives most elaborate instructions as to how the plans were developed. No Greek design has been found which agrees with the Vitruvian statements. In fact, the module would produce a grade of static symmetry which would have afforded much amusement to a Greek.

Dynamic symmetry in nature is the type of orderly arrangement of members of an organism such as we find in a shell or the adjustment of leaves on a plant. There is a great difference between this and the static type. The dynamic is a symmetry suggestive of life and movement.

Its great value to design lies in its power of transition or movement from one form to another in the system. It produces the only perfect modulating process in any of the arts. This symmetry cannot be used unconsciously although many of its shapes are approximated by designers of great native ability whose sense of form is highly developed. It is the symmetry of man and of plants, and the phenomenon of our reaction to classic Greek art and to certain fine forms of other art is probably due to our subconscious feeling of the presence of the beautiful shapes of this symmetry.

Material for the study of dynamic symmetry is obtained from three sources: from Greek and Egyptian art, from the symmetry of man and of plants and from the five regular geometrical solids. We study Greek art for the purpose of learning how the greatest artists have used the rhythmic themes of area in actual masterpieces. The human skeleton shows us how nature employs these same themes in an organism. The five regular solids,* the cube, the tetrahedron, the octahedron, the icosahedron and the dodecahedron furnish abstract geometrical material for study. The skeleton, however, is the source *par excellence* for the artist.

The discovery of dynamic symmetry places the human skeleton in a new position in relation to art. Heretofore it has been but slightly considered—being regarded by artists chiefly as a framework supporting a muscular system of more or less value to them from the anatomical viewpoint. We must now regard this framework of bone as the chief source of the most vital principles of design.

It may not be inappropriate to mention here that the writer's interest lies entirely in the field of creative effort, and that research has been incidental only to his general aim. He was impelled to take up the study of symmetry because he could not entirely agree with the modern tendency to regard design as purely instinctive. As the trend of the individual and of society seems to be toward an advance from feeling to intelligence, from instinct to reason, so the art effort of man must lead to a like goal.

The world cannot always regard the artist as a mere medium who reacts blindly, unintelligently, to a productive yearning. There must come a time when instinct will work with, but be subservient to,

* See Definitions, p. 126.

intelligence. There have been such periods in the past, notably that of classic Greece.

The Greeks were by no means perfect but, for at least a generation in their history, they had a clear understanding of law and order and a passion for its enforcement. Life to them seemed a sublime game, but one meaningless without rules. Indeed, this is the distinction between the Greek and the barbarian.

The Christians struggled for a moral, the Greeks for an intellectual law. The two ideals must be united to secure the greatest good. As moral law without intellectual direction fails, ends in intolerance, so instinctive art without mental control is bound to fail, to end in incoherence. In art the control of reason means the rule of design. Without reason art becomes chaotic. Instinct and feeling must be directed by knowledge and judgment.

It is impossible to correlate our artistic efforts with the phenomena of life without knowledge of life's processes. Without mental control, instinct, or feeling compels the artist to follow nature as a slave a master. He can direct his artistic fate only by learning nature's ideal and going directly for that as a goal.

The present need is for an exposition of the application of dynamic symmetry to the problems of today. The indications are that we stand on the threshold of a design awakening. Returning consciousness is bound to be accompanied by dissatisfaction with the prevailing methods of appropriating the design efforts of the past.

When it is realized that symmetry provides the means of ordering and correlating our design ideas, to the end that intelligent expression may be given to our dreams, we shall no longer tolerate pilfering. Instead of period furniture and antique junk we shall demand design expressive of ourselves and our time. The oriental rug, the style house, the conscious and the artificial craft products, all echoes of other times and other peoples and sure evidence of design poverty, must give place to a healthy and natural expression of the aspirations of our own age.

THE ELEMENTS OF DYNAMIC SYMMETRY

THE DYNAMIC SYMMETRY OF THE PLANT

THE DYNAMIC SYMMETRY OF THE PLANT

The Summation Series.—How Dynamic Symmetry Was Discovered.—The Logarithmic Spiral.—The Law of Phyllotaxis.—Explanation of Its Application to Design.

It has long been known that a peculiar series of numbers is connected with the phenomenon of the orderly distribution of the leaves of plants. The series is: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, etc. This is called a summation series from the fact that each term is composed of the sum of the two preceding terms. For those who may be interested in the subject from the botanical standpoint reference is made to the elaborate work of Professor A. H. Church of Oxford on "Phyllotaxis in Relation to Mechanical Law." This series of numbers in reality represents the symmetry of the plant in the sense that the word is used as "analogy," which is its Greek meaning. From this series we obtain the entire structure of dynamic symmetry, which applies not only to the architecture of the plant but to the architecture of man.

This summation series of numbers, because of its character, represents a ratio, *i.e.*, it is a geometrical progression. This ratio may be obtained by dividing one term into another, such as 34 into 55. The series does not represent the phenomenon exactly but only so far as it is representable by whole numbers. A much closer representation would be obtained by a substitute series such as 118, 191, 309, 500, 809, 1309, 2118, 3427, 5545, 8972, 14517, etc. One term of this series divided into the other equals 1.6180, which is the ratio necessary to explain the symmetry of the plant design system.

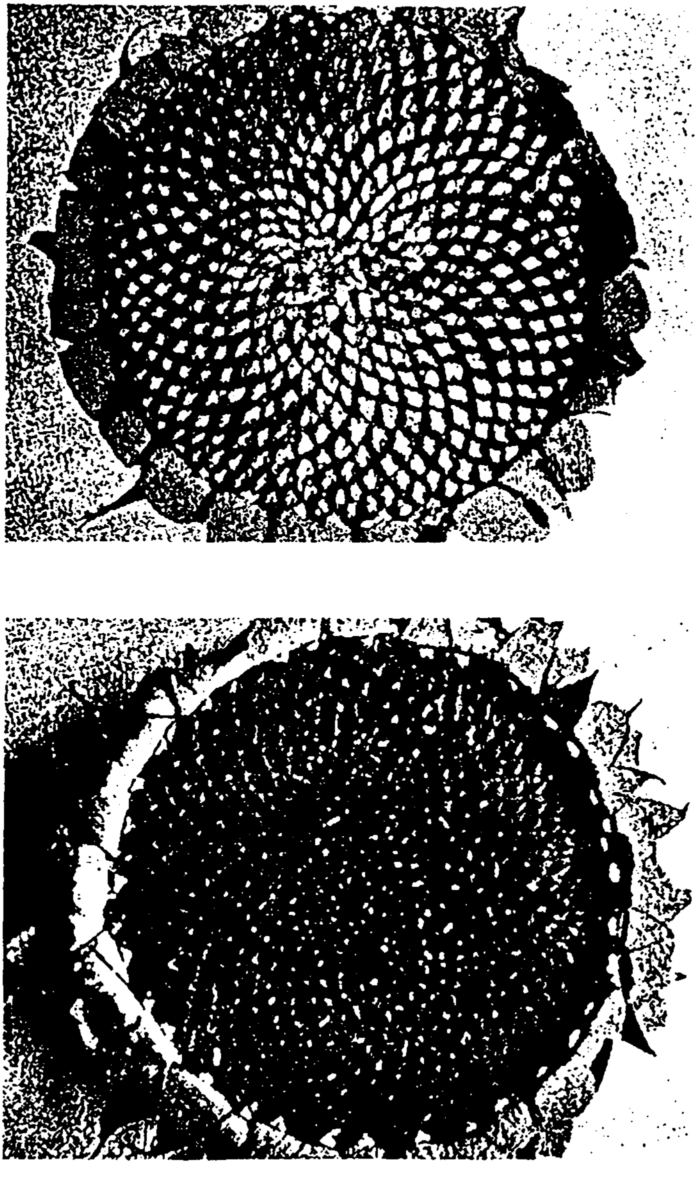
The operation of the law of leaf distribution and its connection with the summation series of numbers is explained by Professor Church, who uses as illustration the disk of the sunflower.

"The most perfect examples of phyllotaxis easily obtainable are afforded by the common sunflower, so frequently selected as a typical Angiosperm, both in anatomical and physiological observations, owing to the fact that it exhibits, *par excellence*, what is regarded as a normal structure little modified by specialization for any peculiar environment. Not only is the sunflower a leading type of the Compositae which holds the highest position among Angiosperm families, but amongst this family it flourishes in the best stations, in which sunlight, air, and water supply are perhaps at an optimum for modern vegetation. The very fact that it is as near an approximation to the typical Angiosperm as can perhaps be obtained, suggests that the phenomena of growth exhibited by it will also be normal, and from the time of Braun to that of Schwendener it has afforded a classical example of spiral phyllotaxis."

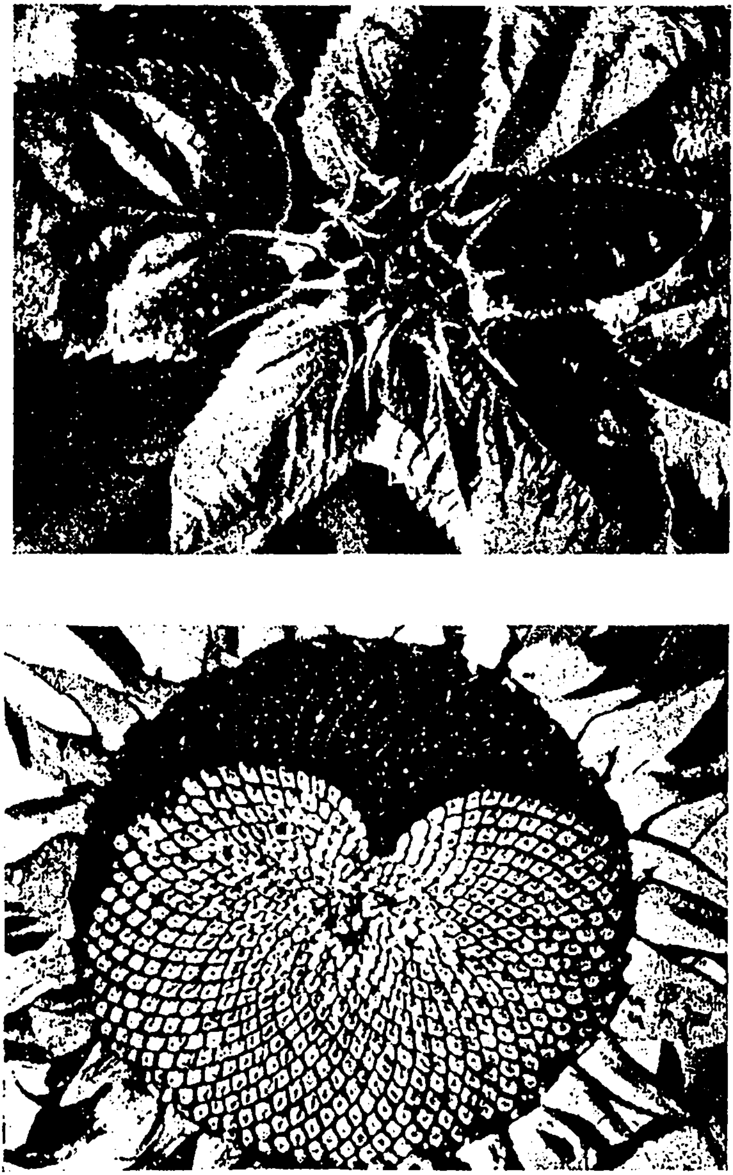
The sunflower heads "admit of ready observation. By taking a head in which the last flowers are withering, and clearing away the corolla tubes, the developing ovaries are seen to mark out rhomboidal facets, and when the fruits are ripened, and have been shed, the subtending bracts still form rhomboidal sockets. These sockets, with or without fruit, form a series of intersecting curves identical with those of the pine cone, only reduced to a horizontal plane.

"A fairly large head, 5–6 inches in diameter in the fruiting condition, will show exactly 55 long curves crossing 89 shorter ones. A head slightly smaller, 3–5 inches across the disk, exactly 34 long and 55 short; very large 11-inch heads give 89 long and 144 short; the smallest tertiary heads reduce to 21–34, and ultimately 13–21 may be found; but these, being developed late in the season, are frequently distorted and do not set fruit well.

"A record head grown at Oxford in 1899 measured 22 inches in diameter, and, though it was not counted, there is every reason to believe that its curves belonged to a still higher series, 144–233. The sunflower is thus limited in its inflorescence to certain set patterns, according to the strength of the inflorescence axis, *e.g.*, 13/21, 21/34, 34/55, 89/144. These were first observed by Braun (1835), and translated into terms of the Schimper-Braun series they would correspond to divergences of 13/34, 21/55, 34/89, 55/144, and 89/233,



Sunflower heads used by Professor Church to exhibit the phyllotaxis phenomena



Sunflower heads used by Professor Church to exhibit the phyllotaxis phenomena respectively. Under normal circumstances of growth, the ratio of the curves is practically constant. (Cf. Weisse. Out of 140 plants 6 only were anomalous, the error being thus only 4 per cent.)"

The ratio 1.6180, when reduced to a rectangular area makes a rectangle which has been given the name by the writer, of the "rectangle of the whirling squares." For a description of a root-five rectangle and the rectangle of the whirling squares, see Lesson 2, Part I.

To the end that the artist may understand the essential idea connected with the form rhythms observable in plant architecture and apply it to his immediate needs, it is advisable to digress at this point and explain how observation of this form rhythm led to the discovery of dynamic symmetry. Many years ago the writer became convinced that the spiral curve found in plant growth, which Professor Church describes in his work on the law of leaf distribution, and that of the curve of the shell, were identical, and must be the equiangular or logarithmic spiral curve of mathematics. It will not be necessary for the present purpose to enumerate all the evidence which justifies this assumption. For the benefit of those who may care to pursue the subject further, reference is made to a paper by the Rev. H. Moseley, "On the Geometrical Forms of Turbinated and Discoid Shell" (Phil. Trans. pp. 351–370, 1838) * and to the able and clear treatment of the spiral by D'Arcy W. Thompson. ("Growth and Form," Cambridge, Eng., 1917.)

Being convinced that the spiral was indeed the mathematical curve mentioned, the writer saw that, because of a certain property which it possessed, this spiral could be reduced from a curve form to one composed of straight lines and thereby be used by the artist to solve certain problems of composition and connect design closely with nature.

This property of the curve is: between any three radii vectors of the curve, equal angular distance apart, the middle one is a mean

*Philosophical Transactions of the Royal Society, London, Vol. 128, 1838, pp. 351–370.

proportional between the other two. The drawing, Fig. I, explains this.

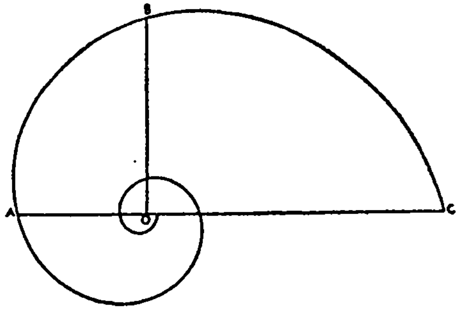


Fig. I
The Logarithmic or Equiangular Spiral

O is the pole or eye of the curve and the lines OA, OB, OC, are radii vectors equal angular distance apart. In this case the angle is a right angle. According to the definition the line OB is a mean proportional between the other two, *i.e.*, OA and OC.

This means, speaking in terms of area, that the line OB is the side of a square equal in area to a rectangle the end and side of which are the lines OA and OC. It necessarily follows, therefore, that if three lines stand in this relationship they constitute the essentials of a right angle. This is shown in Fig. II.

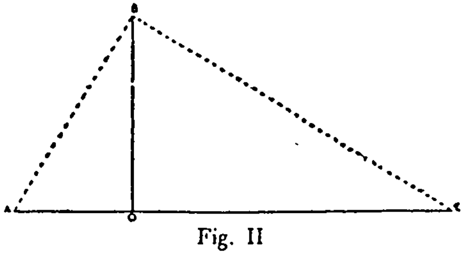


Fig. II

The line OB is a mean proportional between the lines OA and OC and the shape ABC is a triangle, right-angled at B, and the line AC is the hypotenuse.

One of the earliest geometrical facts determined by the ancient Greeks was that in a right angled triangle a line drawn from the intersection of the two sides to meet the hypotenuse at right angles was a mean proportional between the segments of the hypotenuse. Simply stated, this means that three lines situated like the three radii vectors mentioned constitute three terms of a continued proportion; OA is to OB as OB is to OC.

The relation between these lines could be stated arithmetically as a ratio, this following from the fact that lines in continued proportion form a geometrical progression.

If we assume, for example, that the ratio is 2 and the length of the line OA is two units, then the line OB would be four units and the line OC eight units. 2, 4, and 8 constitute three terms of a simple proportion with a ratio of 2 and according to the rule of three, the square of the mean is equal to the product of the extremes. In this case 4 is the mean and 2 and 8 are the extremes. 4 multiplied by itself, or squared, equals 2 multiplied by 8.

The mean is the side of a square equal in area to the area of a rectangle made by the extremes or 2 as end and 8 as side.

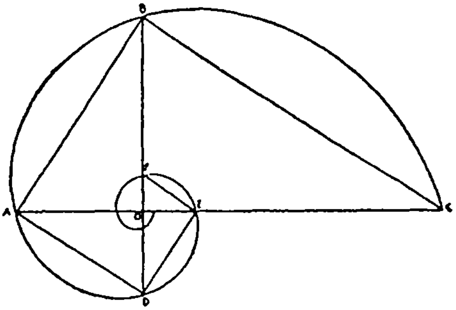


Fig. III

But the line OB may be produced through the pole O of the spiral until it touches the curve at the point D and DAB will be a right angle and there will exist four lines in continued proportion, OD, OA, OB and OC. Also, the curve cuts the vector lines at E and F, etc., and other right angles are created. In short, the curve may be reduced from a curve to a rectangular spiral, and, as in the curve, the converging right angles wrap themselves to infinity around the pole. This particular curve never reaches the pole, it goes on forever.

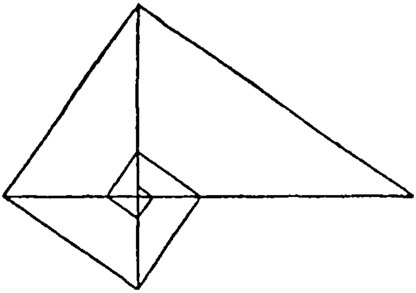


Fig. IV

The essential point is that this simple construction enables the designer to introduce the law of proportion into any type of composition and that, too, in much the same way as it appears in the plant and in the shell. The operation by which this is accomplished is the drawing of a diagonal to a rectangle and a line from one corner cutting this diagonal at right angles, Fig. V.

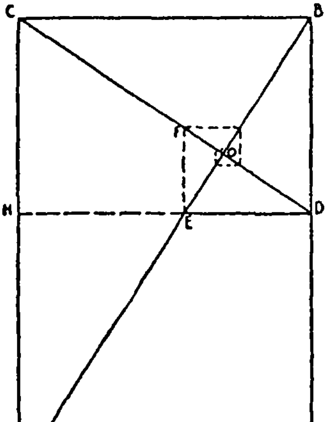


Fig. V

Fig. V is any rectangle and AB is a diagonal. The line CD, drawn from the corner C, cuts the diagonal AB at right angles at O. It is apparent that the angular spiral formed by the lines AC, CB, BD, DE and EF is identical with the angular spiral derived from the shell and from the plant.

Having established a condition of proportion within the area of a rectangle it becomes obvious that the line CD must perform some important function.

Artists are well acquainted with at least one property of the diagonal to a rectangle. It is well known, for example, that any shape drawn within the area of a rectangle whose diagonal is common to the diagonal of the containing area is a similar shape to the whole. This geometrical fact is utilized in the arts, especially the reproductive arts, for the purpose of determining similar shapes on a larger or smaller scale. This, however, is about as far as modern artistic knowledge of proportion extends. The value of the line CD is unknown to modern art. That the value of this line was known at one time in the history of art and its power appreciated is abundantly proven by dynamic analysis of classical Greek design. This line CD determines the reciprocal of a rectangular shape and is itself the diagonal of that shape. Following the construction we appreciate the fact that, in a rectangular area, the diagonal of a reciprocal cuts the diagonal of the whole at right angles. (See Lesson 5.)

The line HD, Fig. V, which is drawn parallel to the ends AG or CB, fixes the area of the reciprocal of the shape AB. With this notion of a reciprocal in mind it is apparent that the fatter or squatter the rectangle the larger will be the area of the reciprocal.

For example, if instead of the rectangle AB of Fig. V, one is constructed which much more closely approaches the shape of a square, the area of the reciprocal, because it is a similar shape to the whole, will also much more closely approach the shape of a square.

The breadth of the reciprocal increases with the breadth of the parent form until it coincides with it as a square; or it decreases until both become a straight line. Obviously, also, there must be rectangles such that the reciprocal is some even multiple of the parent form, such as 1/2, 1/3, 1/4, 1/5 and so on.

Perception of this fact led the writer to discover the root rectangles. It was found that a rectangle whose reciprocal equalled one-half the whole was a root-two rectangle; 1/3 a root-three, 1/4 a root-four and 1/5 a root-five rectangle and so on. When the root-five rectangle was defined and its commensurable area examined it was found that this shape was connected in a curious manner with the phenomena of leaf distribution.

As has been explained, the ratio produced by the summation series of numbers, which so persistently appears in the rhythmic arrangements of leaves and seeds in vegetable growth, is 1.6180. When a rectangle was made wherein the relationship between the end and side was the ratio 1.6180 it was found that the end of the reciprocal of this area equalled .618.

The side of a root-five rectangle, arithmetically expressed, is 2.2360 or the square root of five. If the ratio 1.6180 is subtracted from 2.2360 the remainder is .6180. The area of a root-five rectangle, therefore, is equal to a 1.6180 rectangle plus its reciprocal.

The 1.6180 rectangle, because the end of its reciprocal equals .618, is a rectangle such that its *continued* reciprocals cut off squares and these squares form a spiral around a pole of the rectangle. This pole, of course, is the point where the diagonal of a reciprocal and a diagonal of the whole cross each other. Because of this property this shape was given the name "the rectangle of the whirling squares." (See Lesson 4.)

The importance to design of the curve-cross pattern arrangements found in the leaf distribution phenomena lies in the fact that the normal scheme represented by the sunflower disk is clearly connected with commensurability of area. And, because an area may represent the projection of a solid, the measurableness of the two-dimensional plan is also that of the three-dimensional plant example.

The dynamic rectangles, which we obtain from the growth phenomena, are distinguished by this property of area measurableness. It is this characteristic which lies at the base of the rhythmic theme conception and gives the dynamic scheme its greatest design value.

Because of the persistence of the normal ratios of phyllotaxis the conclusion is inevitable that the measurable area themes possess life and all the qualities that go with it, while areas which do not have this peculiar property do not have life. They are "static" or dead areas, at least as far as design is concerned. If the testimony of Greek art has value this would seem to be so. We know that one characteristic of Greek design is just this life-suggesting quality. Many have noticed it. We know also that Roman art, by comparison, is lifeless. Many have noticed and commented upon this fact. Indeed, we need not go further than Roman sculpture, with its surface commonplaceness and stodgy, uninspired design quality, to see how true this is.

It is recognized that the substitute summation series, which has been suggested to take the place of the whole number series used by botanists to express the curve-cross phenomena of distribution, can have little if any value in checking an actual plant structure. The fractions necessary to make the ratio 1.618 complete for the system are so small that their presence in a specimen could not be noted.

We are using the phyllotaxis phenomena for purposes of design and are not so much interested in botanical research. The plant structure is so delicate and the skeletal residue so slight, after an average plant withers, that direct observation for commensurable area themes is out of the question. We may leave the field to the botanist and let him retain the conventional series of whole numbers.

To the artist, however, the numbers of the substitute series, 118, 191, 309, 500, 809, 1309, 2118, 3427, 5545, etc., are of much interest, because they not only furnish the exact ratio of 1.6180, but each member of the series is an actual ratio of the dynamic commensurable area scheme and is found abundantly not only in the human skeleton, but throughout classic Greek design.

Nature provides many plant examples wherein the curve-cross system varies from the normal whole number series. These less common arrangements of curve-cross pattern form are mentioned by Church in the following terms:

"When the type of normal asymmetrical phyllotaxis is thus completely isolated as consisting of systems mapped out by log spiral curves in the ratio-series of Braun and Fibonacci, 2, 3, 5, 8, etc.; and the type of normal symmetrical phyllotaxis is equally clearly delimited as a secondary construction, physiologically independent of the ratio-series, though connected with it phylogenetically, the greatest interest attaches to all other phyllotaxis phenomena, which, though less common, may throw light on the causes which tend to induce symmetry, before postulating, as a last resource, some hypothetical inherent tendency in the protoplasm itself.

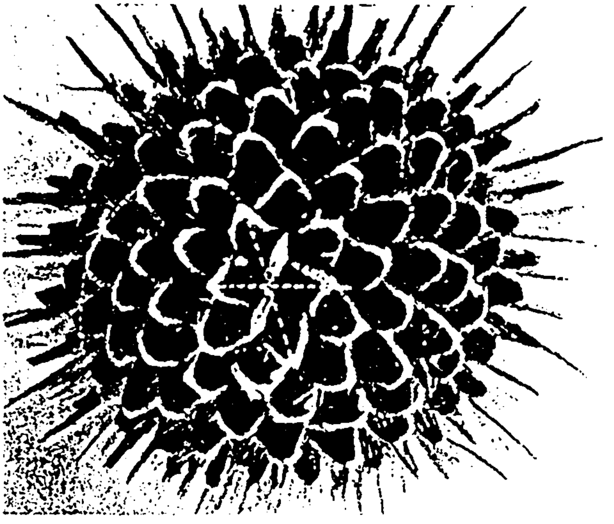
"These may be included under two series: first, the multijugate systems of Bravais, and, second, systems in which the parastichy ratios belong to series other than that of Braun and Fibonacci, *e.g.*, the 3, 4, 7, 11———, 4, 5, 9, 14———, or still higher series (these latter Church classifies as belonging to anomalous series. It will be noticed that they also are summation series).

"The term *multijugate* was applied by the brothers Bravais to types of phyllotaxis in which the numbers expressing the parastichy ratios are divisible by a common factor; so that 2 (13/21) = (26/42), a bijugate system; while 3 (13/21) = (39/63) would be a trijugate one."

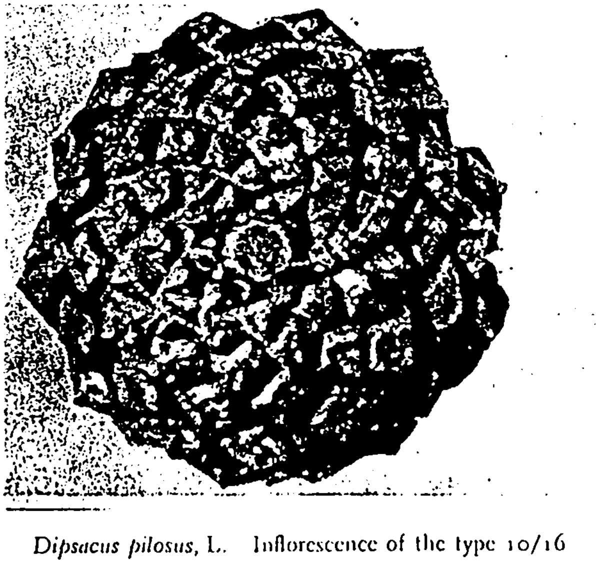
The fact that they may occur in the plant (*i.e.*, multijugate systems) which has already been found to exhibit normal phyllotaxis phenomena most completely, lends additional interest to these constructions. Thus, out of a batch of heads, collected at haphazard by E. G. Broome, two were bijugate, 26/42 (read 26 crossing 42) and 42/68, respectively; the others were quite normal; while out of a total crop of 130 cones on a plant of *pinus pumilis* one cone only was 6/10/16—see illustration—the rest being normal, or 5/8/13, "Phyllotaxis in Relation to Mechanical Law."

These so-called multijugate types are interesting, particularly when a specimen shows a variation from the normal scheme to one of multiples of that ratio by 2 or 3. Using the suggested substitute series and considering the commensurable area themes of dynamic symmetry, the bijugate scheme as 26 crossing 42 is 1309 crossing 2118 wherein both members are multiplied by 2.

The trijugate arrangement, 39 crossing 63, becomes 1309 and 2118 multiplied by three, and the schemes would be, as measurable area arrangements, 2618, 4236, 3927, 6354, respectively. A 6/10/16 plan,



Pinus pumilis, bijugate cone of type 6/10/16



Dipsacus pilosus, L. Inflorescence of the type 10/16 as a commensurable area theme of dynamic symmetry would be .618, 1. and 1.618.

This leads us to consider somewhat in detail the substitute summation series. Its properties are curiously interesting. Any two members of the series added equal a third member.

Every term divided into its successor equals the ratio 1.618.

Every term divided into a third term equals 1.618 squared or 2.618.

Every term divided into a fourth term equals 1.618 cubed or 4.236.

Every term divided into a fifth term equals 1.618 raised to the 4th power or 6.854.

Every term divided into a sixth term equals 1.618 raised to the 5th power or 11.090, etc.

Powers of 1.618 divided by 2 produce the summation series of numbers.

$\frac{2618}{2}$ equals 1309

$\frac{4236}{2}$ equals 2118

$\frac{6854}{2}$ equals 3427

$\frac{11090}{2}$ equals 5545

$\frac{17944}{2}$ equals 8972

$\frac{29034}{2}$ equals 14517

$\frac{46978}{2}$ equals 23489, etc.

1.618 divided by 2 equals .809.

Unity divided by 2 equals .5.

.618 squared equals .382 $\frac{.618}{2}$ equals .309

.618 cubed equals .236 $\frac{.382}{2}$ equals .191

.618 4th. power equals .146, etc. $\frac{.236}{2}$ equals .118, etc.

The square of 1.618 equals 1.618 plus 1. or 2.618.

.618 plus .618 squared equals 1.

The student will recognize all these numbers as ratios representing the dynamic measurable areas. The few relationships mentioned are merely suggestive; an extraordinary number of curiously fascinating interdependencies may be worked out of this summation series. When the series is charted as the expansion of a scheme of squares and subdivisions of squares it appears as in Fig. VI.

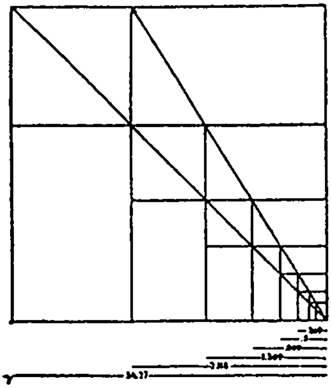


Fig. VI

It will be clear to those who have studied dynamic symmetry that the basic pattern plan of leaf arrangement is but one phase of an architectural scheme connected with the phenomena of growth in general. Fortunately, as artists, we need not depend upon plant forms for detailed knowledge of the mysteries of dynamic form. The human skeleton furnishes all that the plant does and more. It is practically impossible to measure the withered plant structure. Comparatively, the bone network of man's frame is more stable than the skeleton of the plant, and, what is of much greater artistic importance, man is and always has been recognized as the most perfect type of design in nature.

It is felt that the foregoing rough outline of the dynamic symmetry of the plant is all that is necessary to show the artist that the dynamic scheme has a general basis in plant growth.