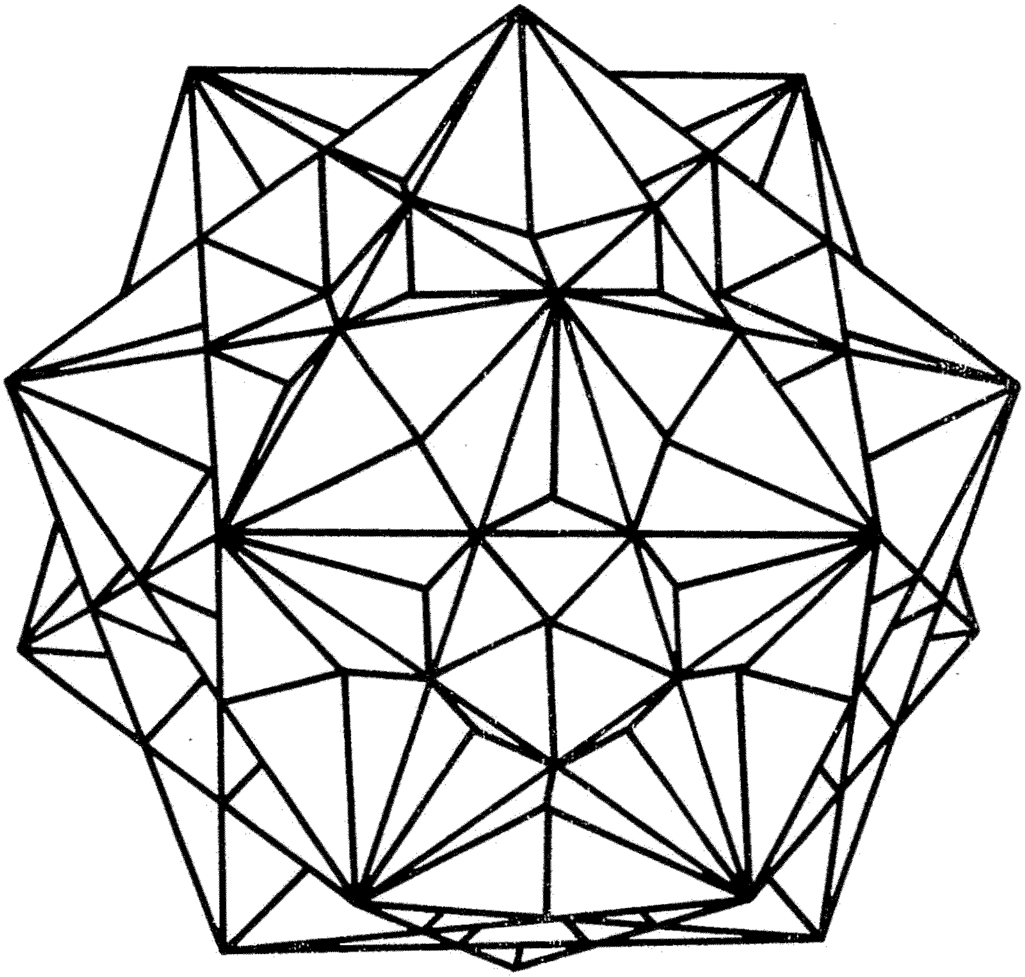
col. r.s. beard



patterns in space

ABOUT THE AUTHOR

By any standard, Robert Stanley Beard is a remarkable man. Since graduating from M.I.T. in 1905, he has been a successful railroad engineer, city engineer, career Army engineer, and creator of beautiful geometrical designs. At the age of 90, his hand is steady, his eyes keen, and his sense of humor acute. Scarcely a day goes by but that he devotes several hours to plane or solid geometrical problems with model making a specialty. For nearly three decades he has shared his collections of polyhedra with the mathematical community. One set of these models is on permanent display at Teachers College, Columbia University and another at the University of California at Berkeley. He often appears at professional meetings and has authored some thirty papers on various aspects of design.

The present volume is truly unique. At least I have found it so, since it is the only book that I own which captivated my daughter at 7 and my father-in-law at 77! Not only does Colonel Beard show us some unusual designs, but he also tells us how the constructions were made. Such material is especially refreshing since so little in modern school curricula emphasizes the esthetical and constructional aspects of geometric figures.

After scanning this book for the first time, I was reminded of Edna St. Vincent Millay's poem which begins: "Euclid alone has looked on Beauty bare." Perhaps if she had known R. S. Beard, she would have modified that line ever so slightly.

John D. Hancock
California State University
Hayward, California

PREFACE

Much like Uncle Tom's Topsy, this book 'was never born. It just growed.' It grew under the notion that each of us should keep helping things along as best he can to justify our existence. Way back, Plutarch said that the mind is not a vessel to be filled but a fire to be ignited. Shortly after the Second World War left me benched for age, much was being said about geometry being the most disliked of high school studies in spite of its importance in the education of scientists and engineers. Even President Hoover had made some comments about this situation.

Geometry had been a prime factor in my existence ever since the horse and buggy days of the Spanish American War. These disparaging remarks struck a sympathetic chord in my genes and spurred me to explore what might be done to generate some internal radiation in the minds and hearts of students of geometry.

Work along these lines created an opportunity in 1950 to join in organizing a mathematical laboratory for the Institute of Mathematics of Yeshiva University at the invitation of Dr. Jekuthiel Ginsburg, head of the Institute and Editor of *Scripta Mathematica.* The drawings in this book are largely an outgrowth of my five years of conducting that laboratory, the first years jointly with Dr. Hermann von Baravalle.

In 1952 I constructed an elaborate set of polyhedra as my assignment on an ad hoc committee to organize a New York City Mathematical Museum. These models became a permanent exhibit at Teachers College of Columbia University. A second more elaborate set of polyhedra has since been constructed for a permanent exhibit at the Lawrence Hall of Science, University of California, Berkeley.

This, then unfinished, book was first assembled in 1963 with the advice and encouragement of Jens L. Lund, Supervisor of the Teaching of Mathematics, School of Education, University of California, Berkeley, and Brother Alfred Brousseau of St. Mary's College and Managing Editor of the Fibonacci Quarterly. Both of these men are former presidents of the California Mathematics Council.

Dale G. Seymour, President of Creative Publications, has been interested in this project from its California beginnings. He has published much of the material in several of his books and also in the form of colored posters based on individual drawings. Creative Publications is now editing the book, adding a number of my new drawings and indexing the book for publication in permanent form.

Col. Robert S. Beard

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section 1

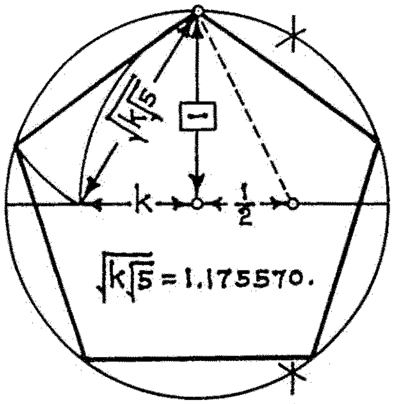
POLYGONS

DRAWING THE POLYGONS.

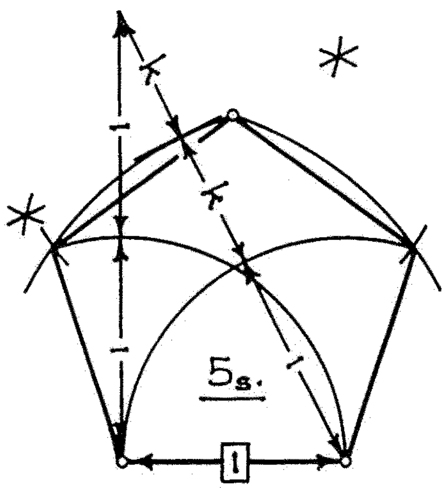
The figures numbered 5_R, 5_S and 5_W show how to draw regular pentagons when radius, R, of circumscribing circle, side, S, or width, W, of pentagon is given. $k=\frac{1}{2}\sqrt{5}-\frac{1}{2}=0.618034$.

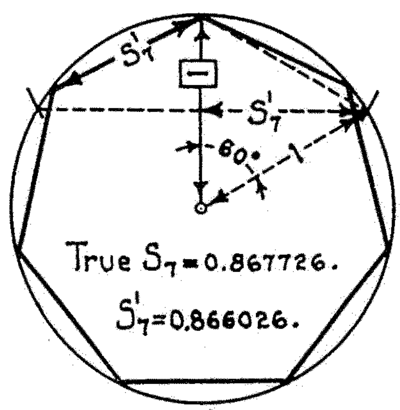
Given dimensions are made unity and are marked thus: ←[1]→

In prime numbered figures, such as $7'_R$, the construction is a close approximation.

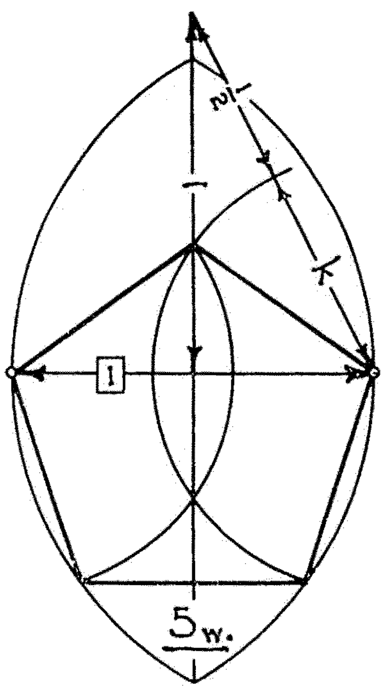


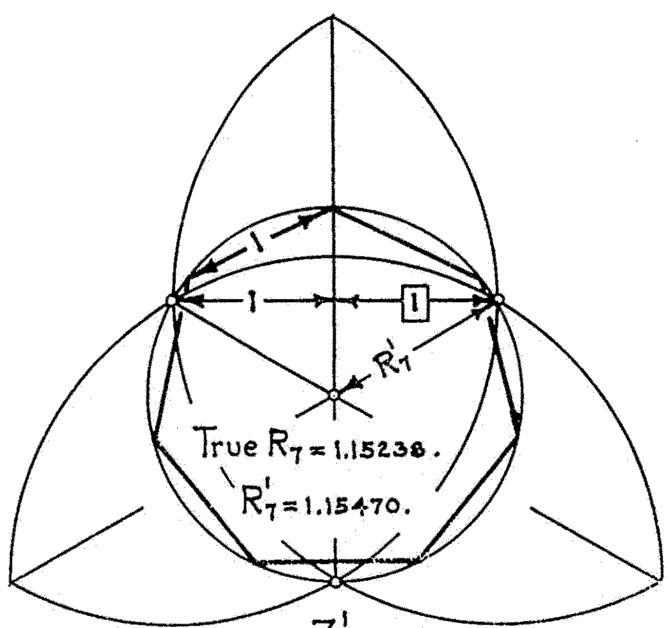
5_R.





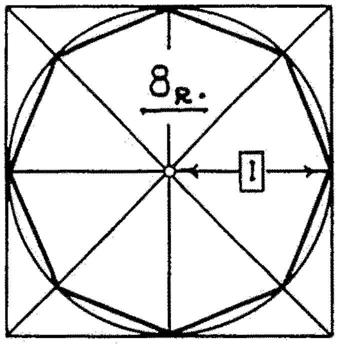
$7'_R$.

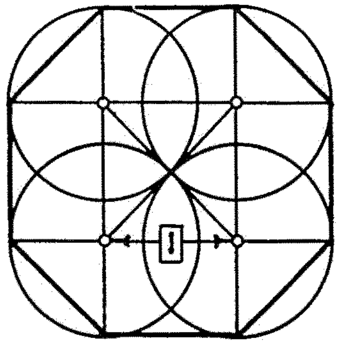




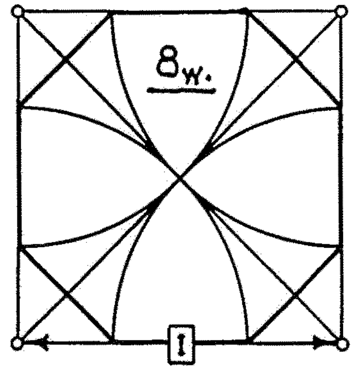
$7'_S$.

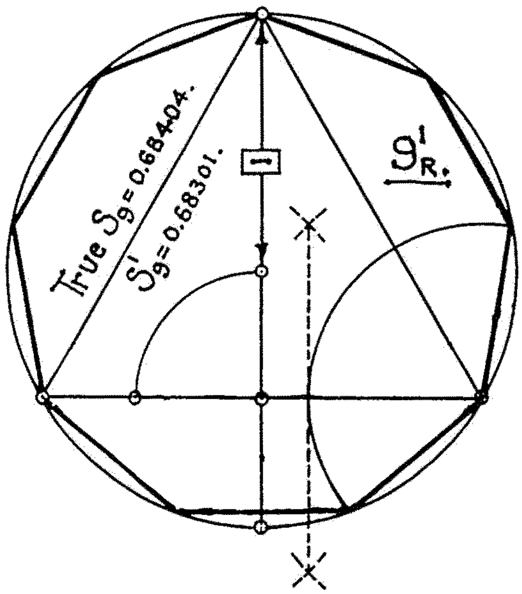
DRAWING THE POLYGONS.

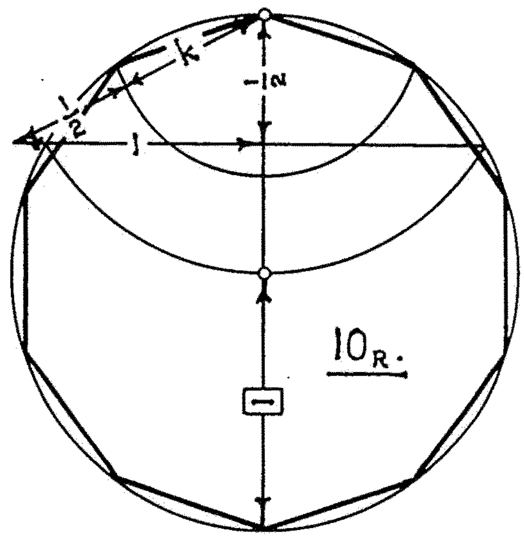


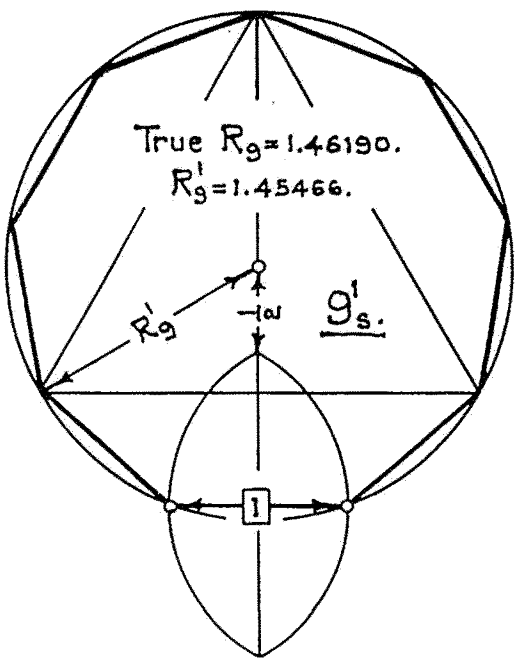


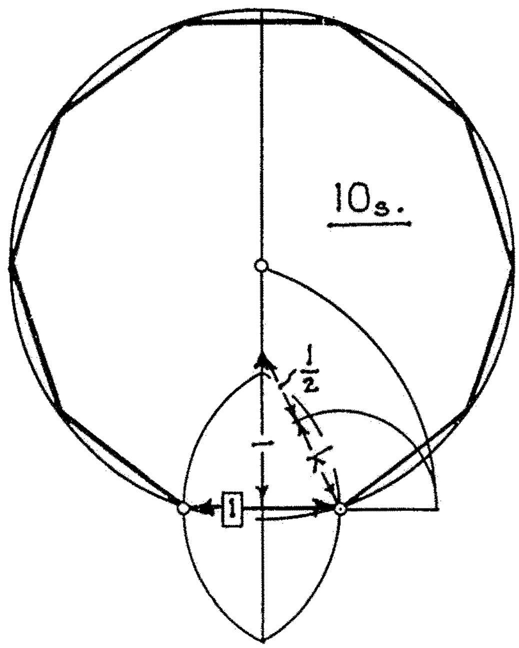
8_S.



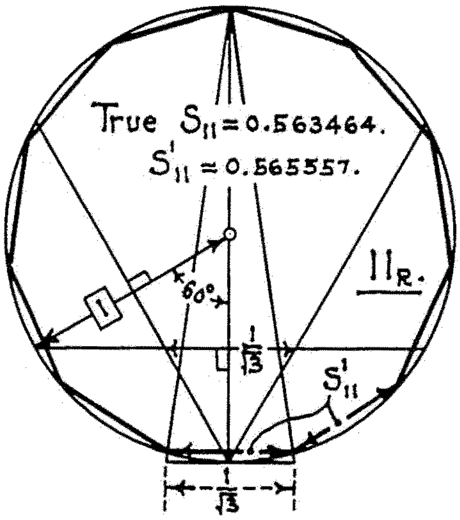


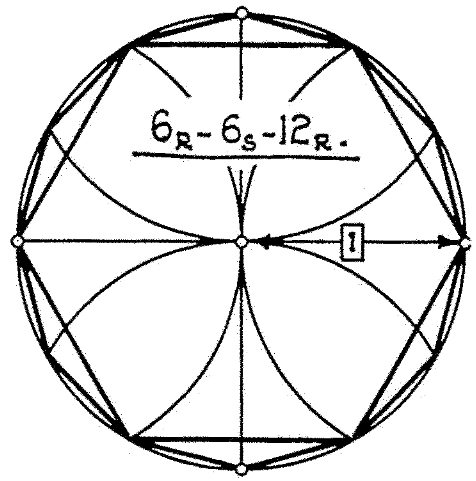


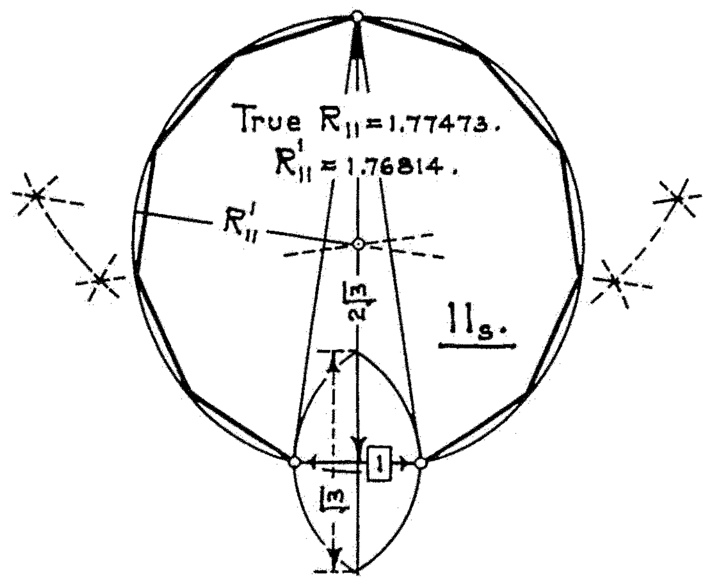


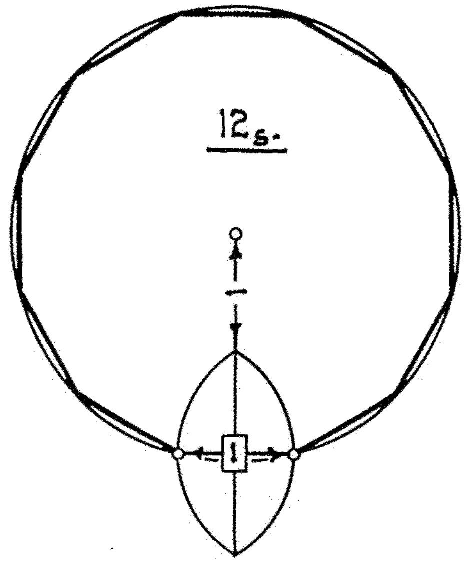


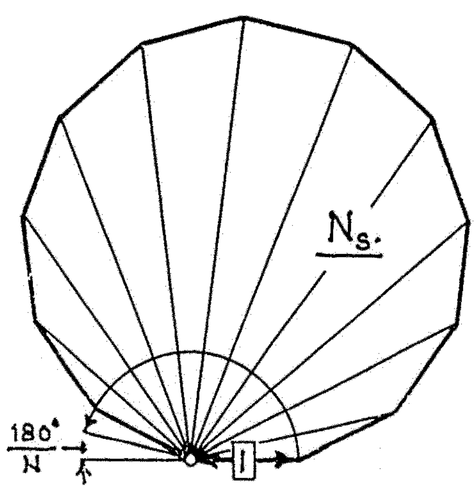
Drawing the Polygons.

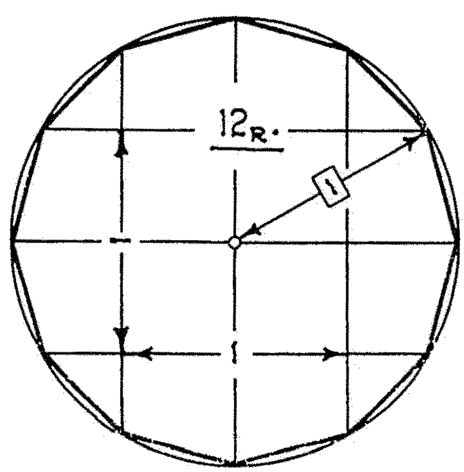












Drawing the Polygons.

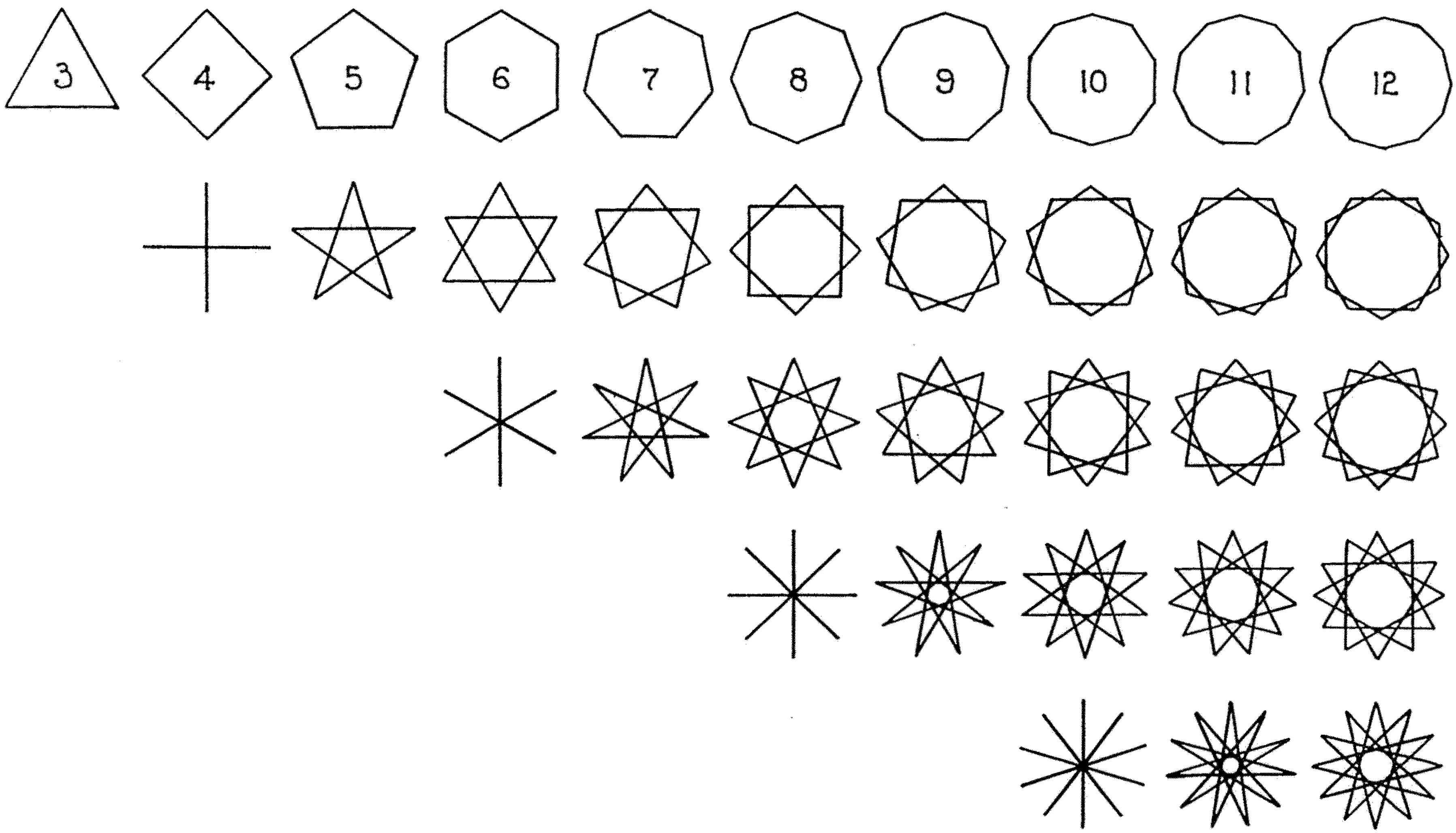
An n sided regular polygon with sides of a given length, s, can be inscribed in a circle of xs radius.

A circle of given radius, r, circumscribes an n sided regular polygon with sides of yr length.

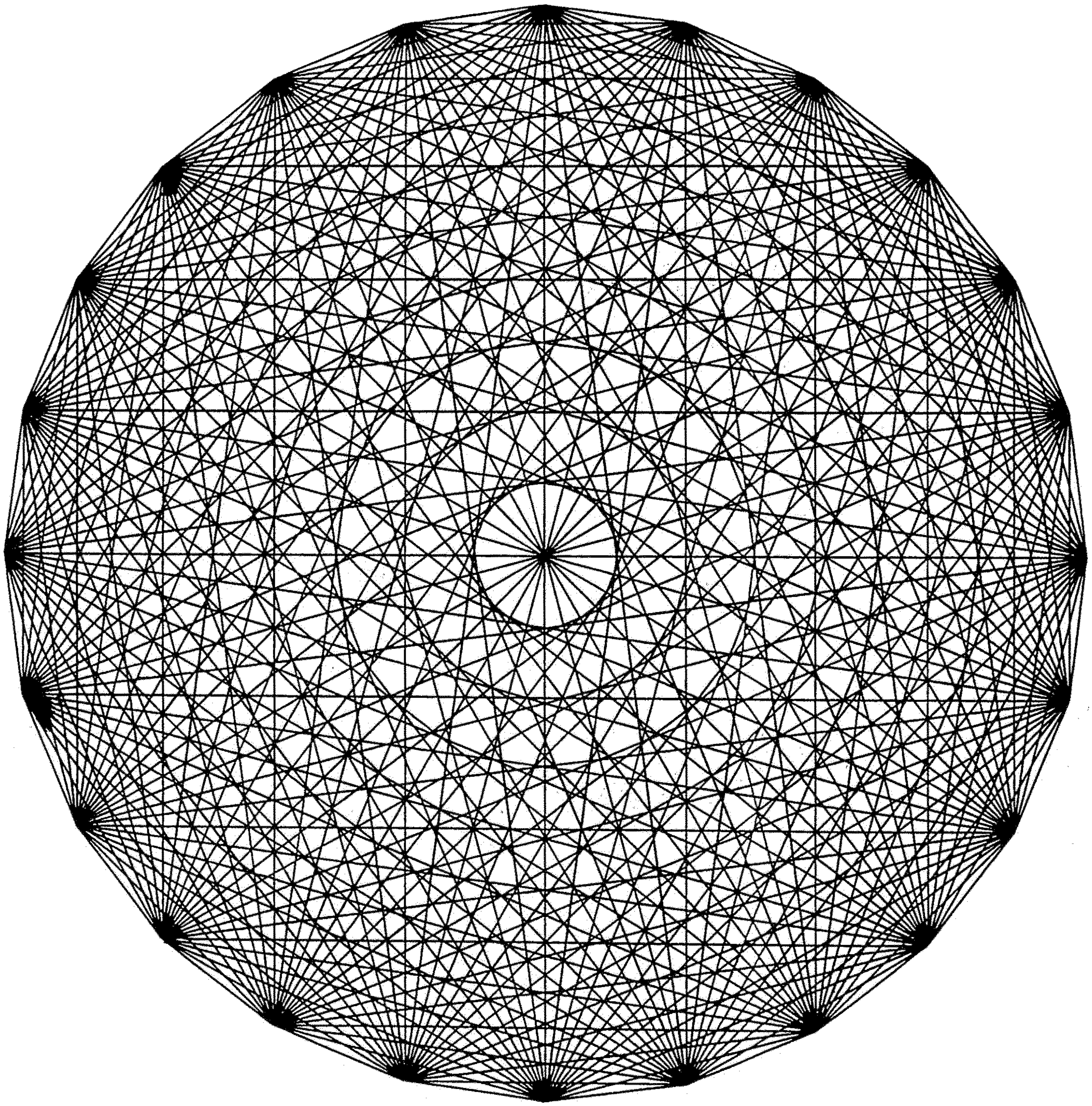
RSB.

n	$\frac{360^\circ}{n}$	x	y	n	$\frac{360^\circ}{n}$	x	y
3	120°	0.5774	1.732	20	18°	3.196	0.3129
4	90°	0.7071	1.414	21	17°09'	3.355	0.2981
5	72°	0.8506	1.176	22	16°22'	3.513	0.2846
6	60°	1.0000	1.0000	23	13°59'	3.672	0.2723
7	51°26'	1.1525	0.8678	24	15°	3.831	0.2611
8	45°	1.3066	0.7654	26	13°51'	4.148	0.2411
9	40°	1.4619	0.6840	28	12°51'	4.468	0.2239
10	36°	1.6180	0.6180	30	12°	4.783	0.2091
11	32°44'	1.7747	0.5635	32	11°15'	5.101	0.1960
12	30°	1.9318	0.5176	34	10°35'	5.419	0.1845
13	27°42'	2.089	0.4786	36	10°	5.737	0.1743
14	25°43'	2.247	0.4450	42	8°34'	6.694	0.1495
15	24°	2.405	0.4158	48	7°30'	7.645	0.1308
16	22°30'	2.563	0.3902	60	6°	9.554	0.1047
17	21°11'	2.721	0.3675	72	5°	11.462	0.0872
18	20°	2.879	0.3473	84	4°17'	13.372	0.0748
19	18°57'	3.038	0.3292	96	3°45	15.282	0.0654

Diagonals of Regular Polygons.

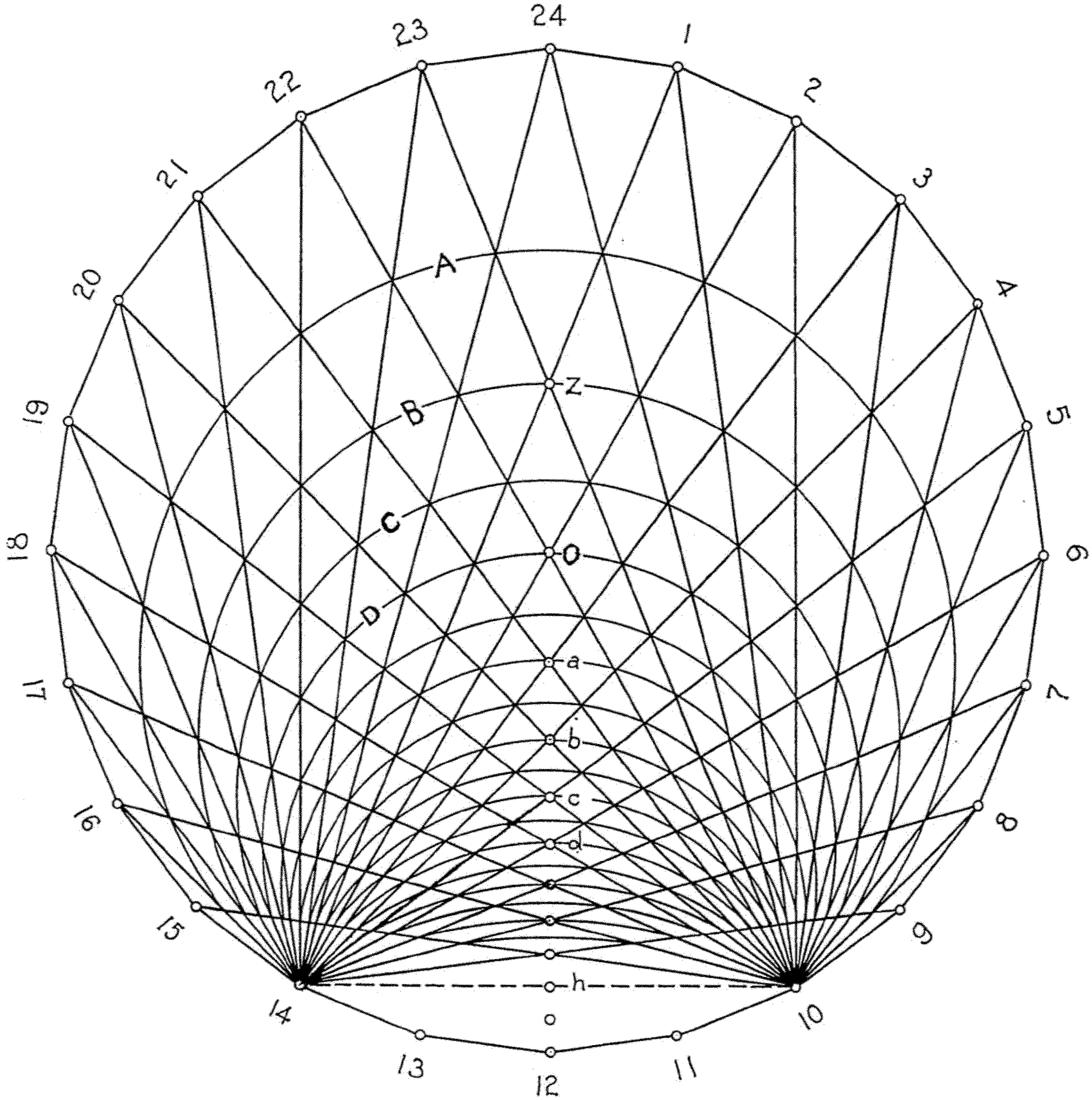


Diagonals of 24 Sided Polygon.



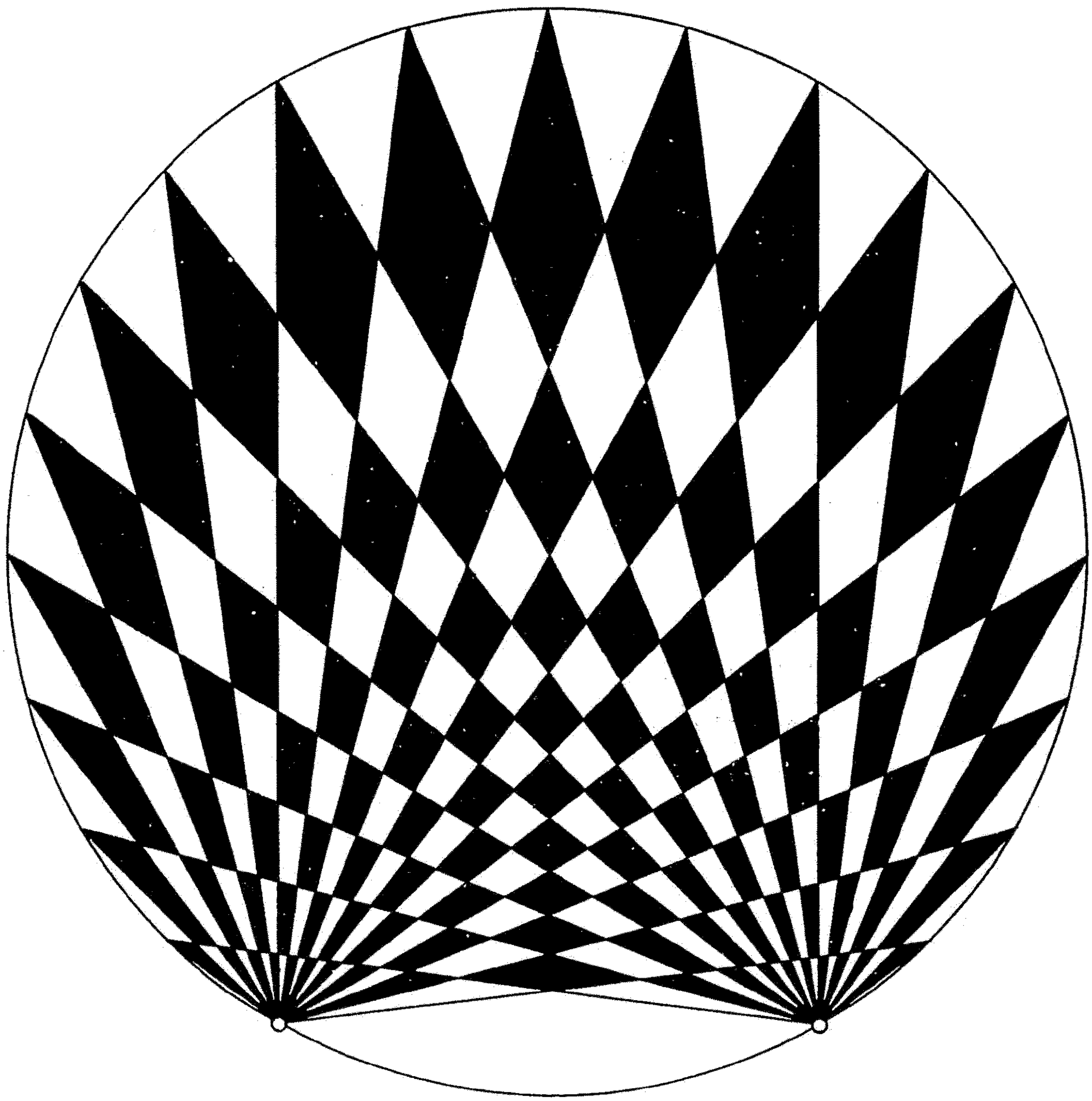
Two Sets of Diagonals in 24 Sided Polygon.

The diagonals that intersect on circle A intercept one side of the bounding polygon. The diagonals that intersect on circle B intercept two sides of the polygon. And so on. Points a,b,c,d,etc are the centers of circles A,B,C,D,etc respectively. The centers of the last nine circles lie on the vertical axis below point h in the reverse order of points h to z.

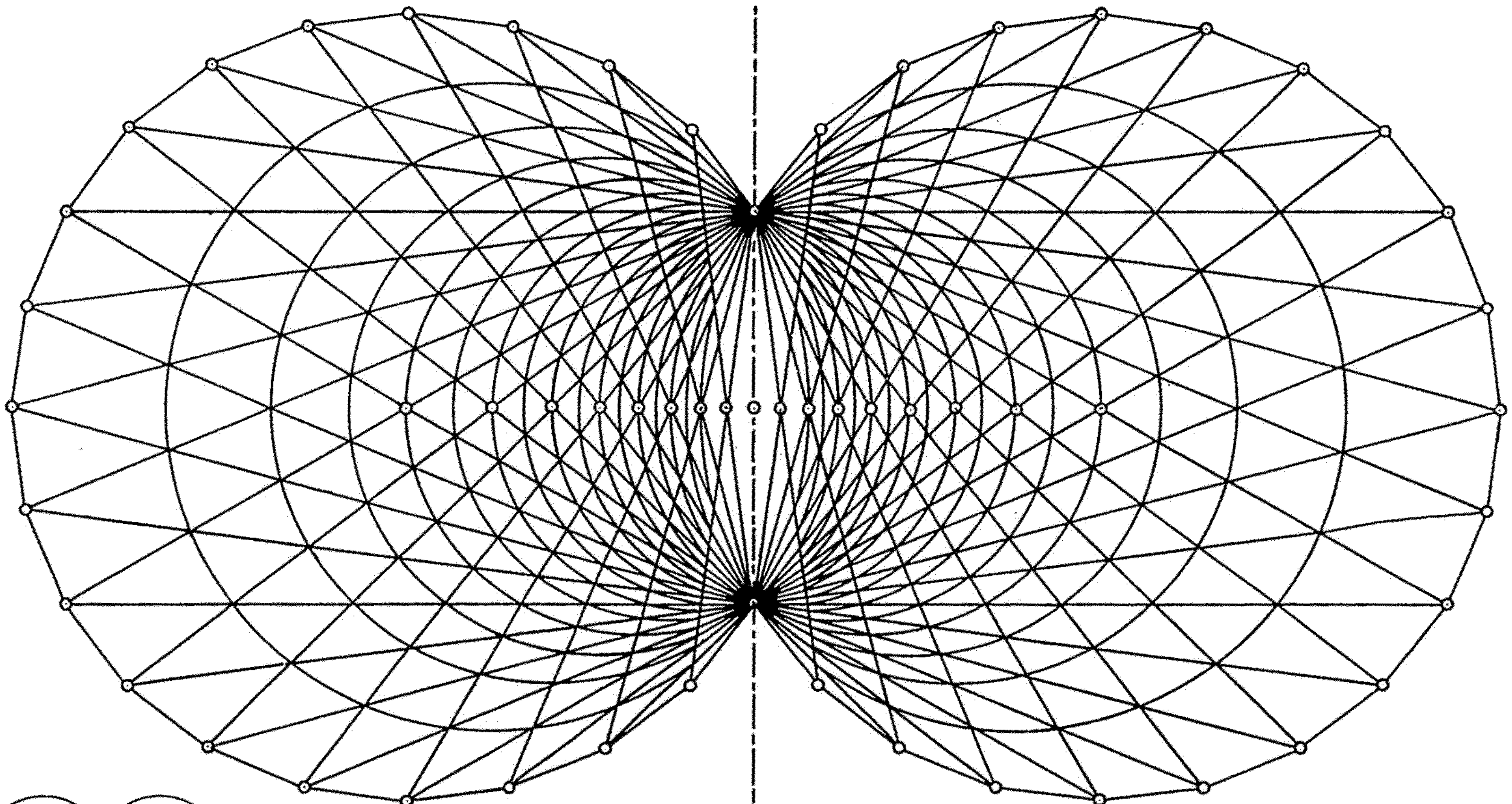


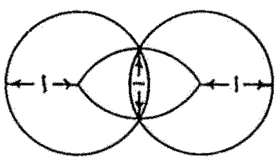
Series.– Diagonals in 24 Sided Polygon.

This tessellation transforms a line drawing into a surface effect.
Treating the drawing of the full set of diagonals as a projection of the earth on its equatorial plane sets the poles at its center. The $7\frac{1}{2}°$ circles of latitude lie in its concentric polygons. Its diameters become the circles of longitude at 15° intervals.
A multicolored drawing of this polygon with all diagonals can have a separate color for each interior polygon if diagonals of the same length are colored alike.



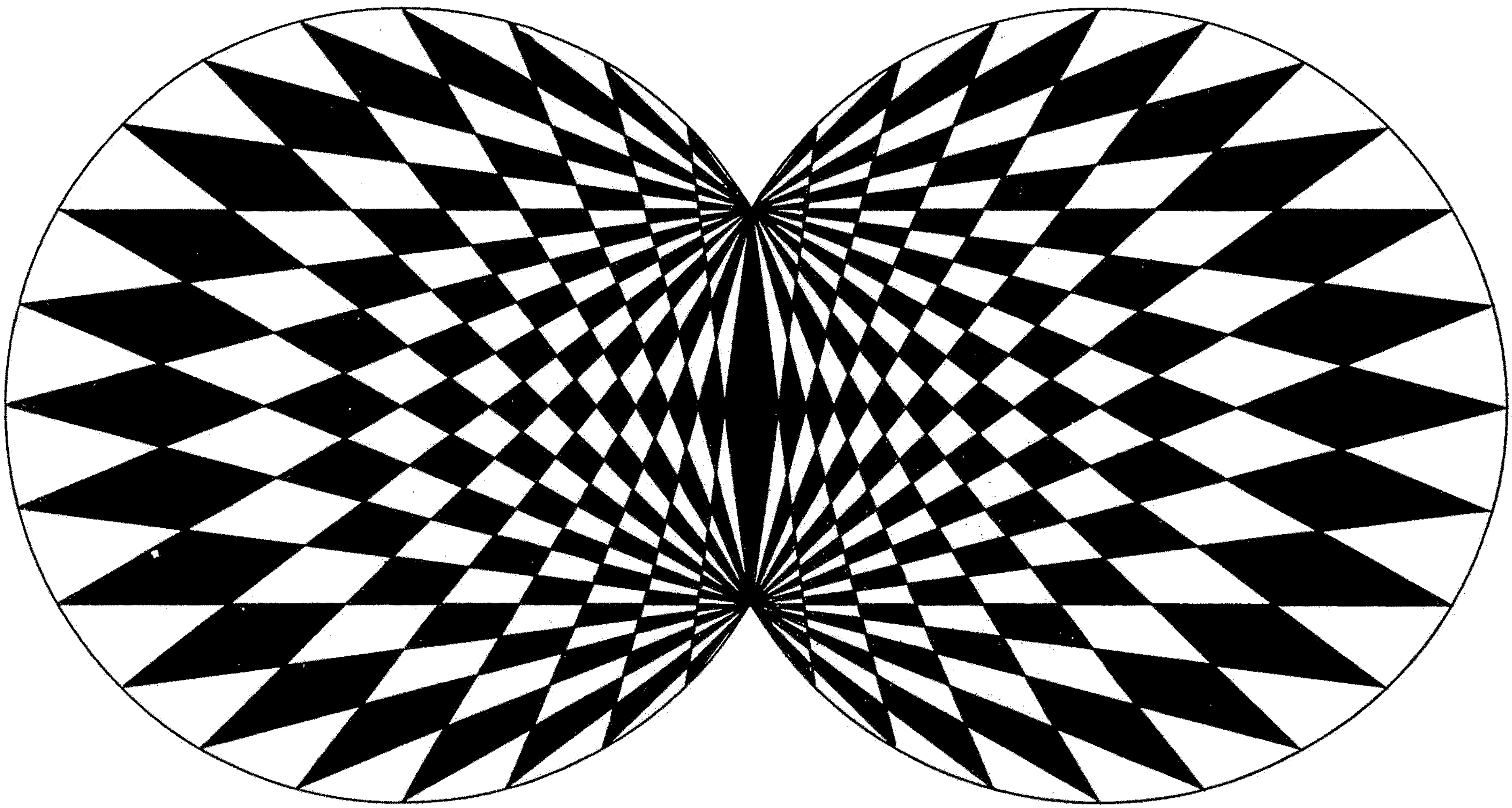
Two Sets of Diagonals in 24 Sided Polygon in Duplicate.





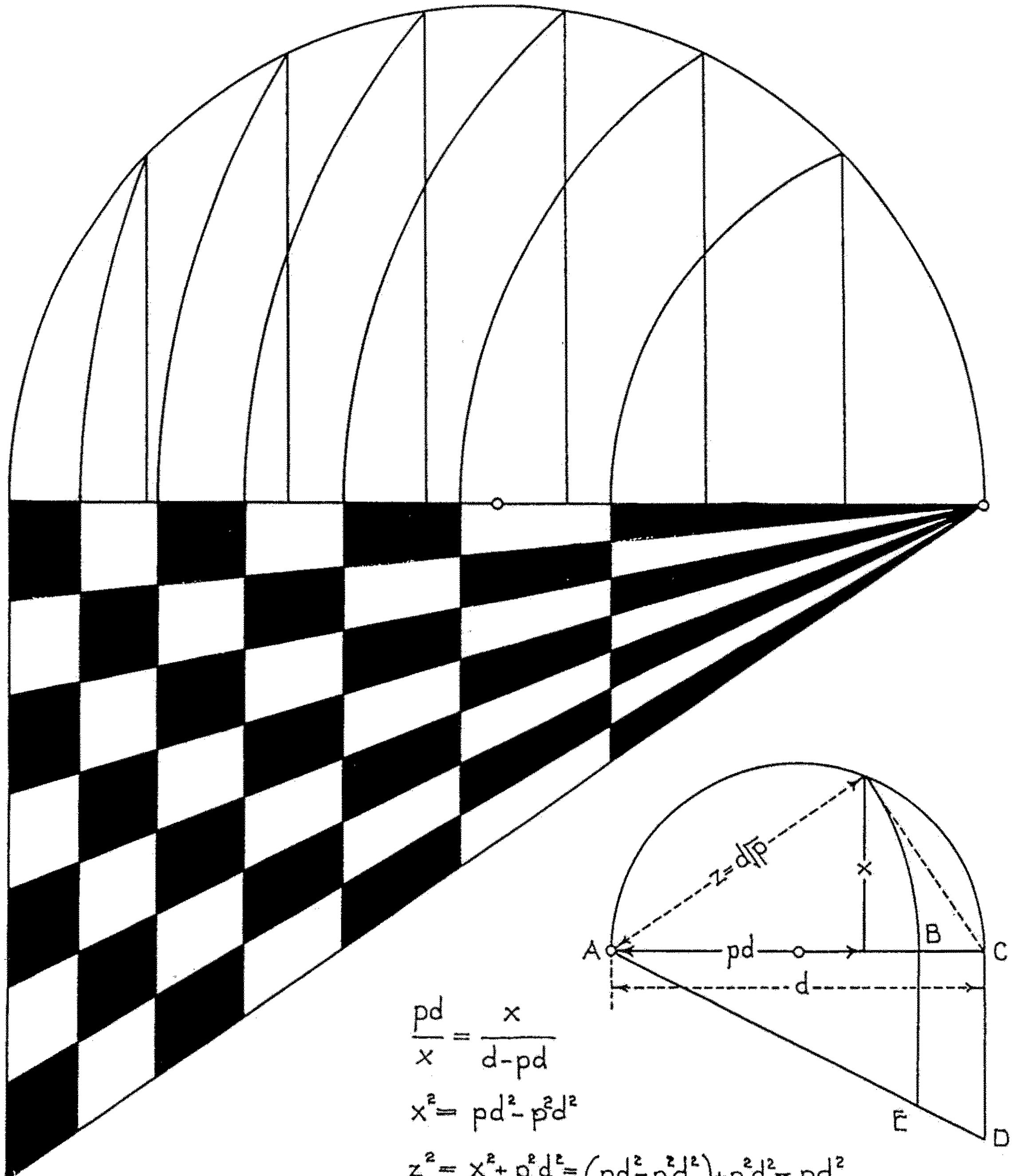
The radii and common chord of the bounding circles are equal.

Each encircled interior point is the center for a circular arc in each half of this twin figure.



A Right Triangle Cut into n^2 Equal Parts.

Divide each leg of triangle into n equal parts and follow construction shown here for $n = 7$.



$$\frac{pd}{x} = \frac{x}{d-pd}$$

$$x^2 = pd^2 - p^2d^2$$

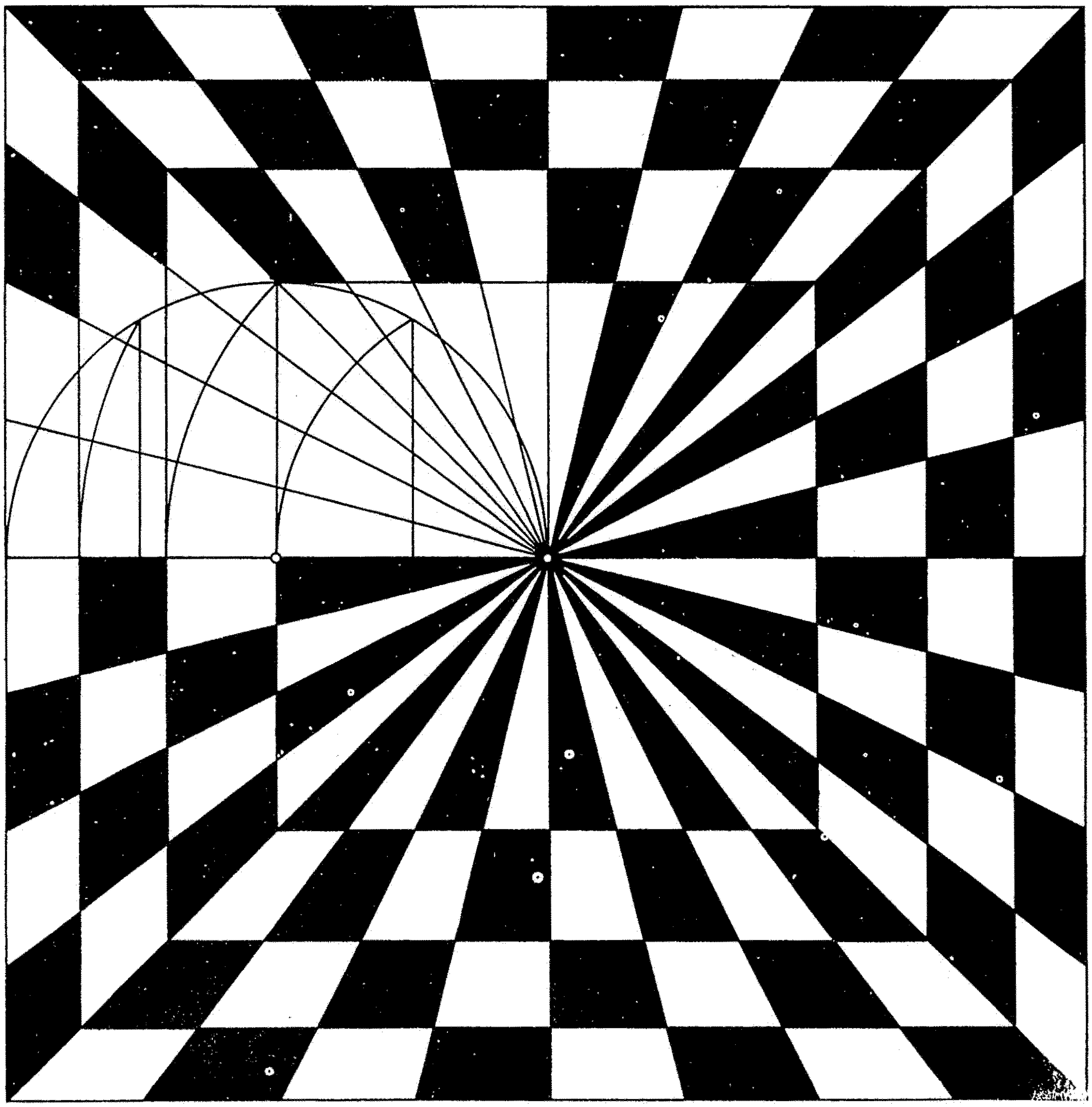
$$z^2 = x^2 + p^2d^2 = (pd^2 - p^2d^2) + p^2d^2 = pd^2$$

$$z = d\sqrt{p}$$

$$\frac{\text{Area } \triangle ABE}{\text{Area } \triangle ACD} = \frac{(d\sqrt{p})^2}{d^2} = p$$

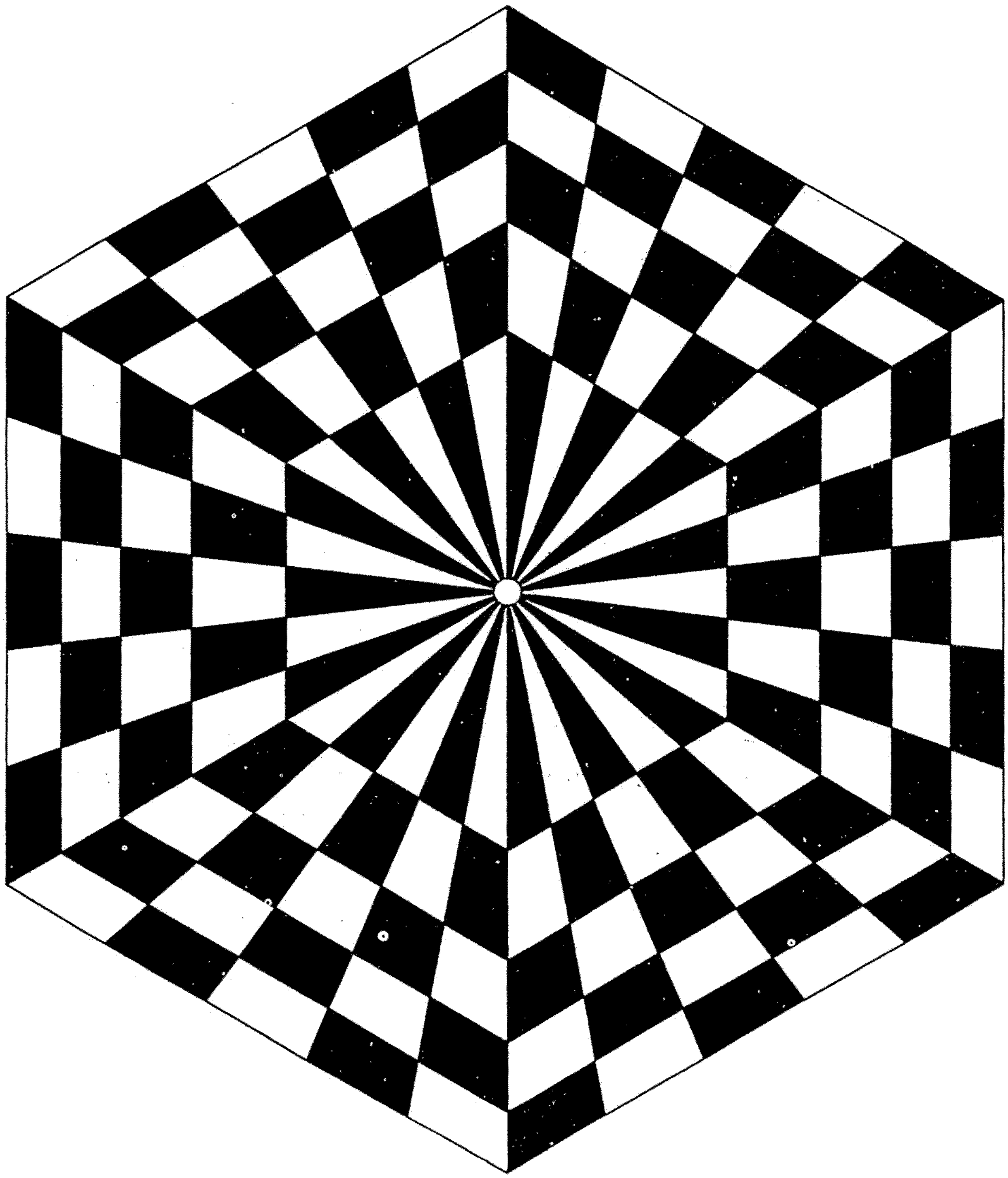
Subdividing a Square.

The interior squares divide the figure into four equal areas.
All unit spaces have equal areas.
Use this construction on the altitude of any triangle.

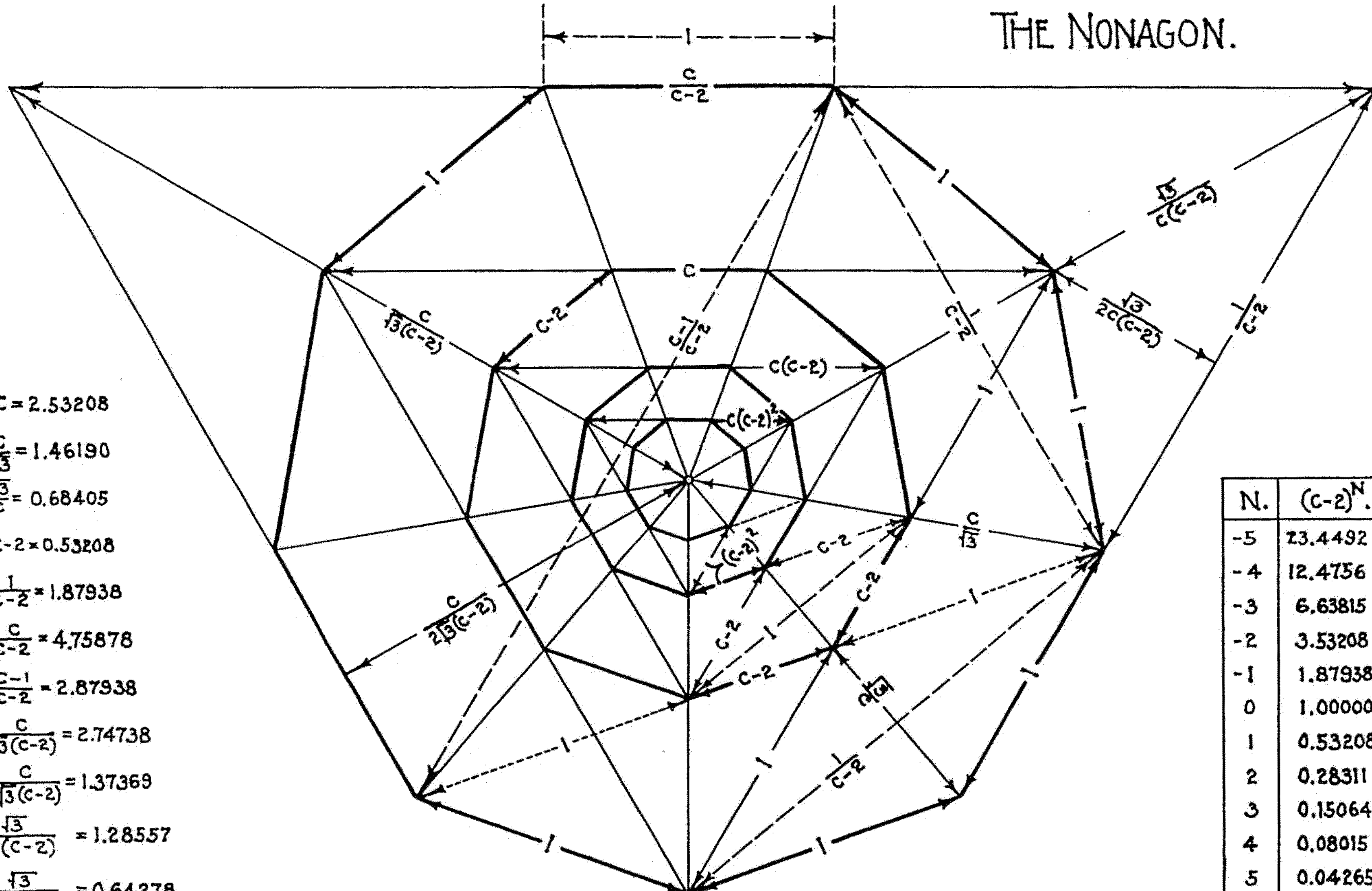


Five Concentric Hexagons.

Ratio of sides of hexagons. $1 - \sqrt{2} - \sqrt{3} - \sqrt{4} - \sqrt{5}$.
Ratio of areas of hexagons. 1 - 2 - 3 - 4 - 5 .
All unit spaces have equal areas.



THE NONAGON.



$c = 2.53208$

$\frac{c}{\sqrt{3}} = 1.46190$

$\frac{\sqrt{3}}{c} = 0.68405$

$c-2 = 0.53208$

$\frac{1}{c-2} = 1.87938$

$\frac{c}{c-2} = 4.75878$

$\frac{c-1}{c-2} = 2.87938$

$\frac{c}{\sqrt{3}(c-2)} = 2.74738$

$\frac{c}{2\sqrt{3}(c-2)} = 1.37369$

$\frac{\sqrt{3}}{c(c-2)} = 1.28557$

$\frac{\sqrt{3}}{2c(c-2)} = 0.64278$

N.	$(c-2)^N$.
-5	23.4492
-4	12.4756
-3	6.63815
-2	3.53208
-1	1.87938
0	1.00000
1	0.53208
2	0.28311
3	0.15064
4	0.08015
5	0.04265

THE NONAGON.

① $\left(\frac{c}{2}-\frac{1}{2}\right)^2+\left[\sqrt{\frac{c^2}{3}-\frac{1}{4}}-\frac{c}{2\sqrt{3}}\right]^2=1.$ Ⓐ.

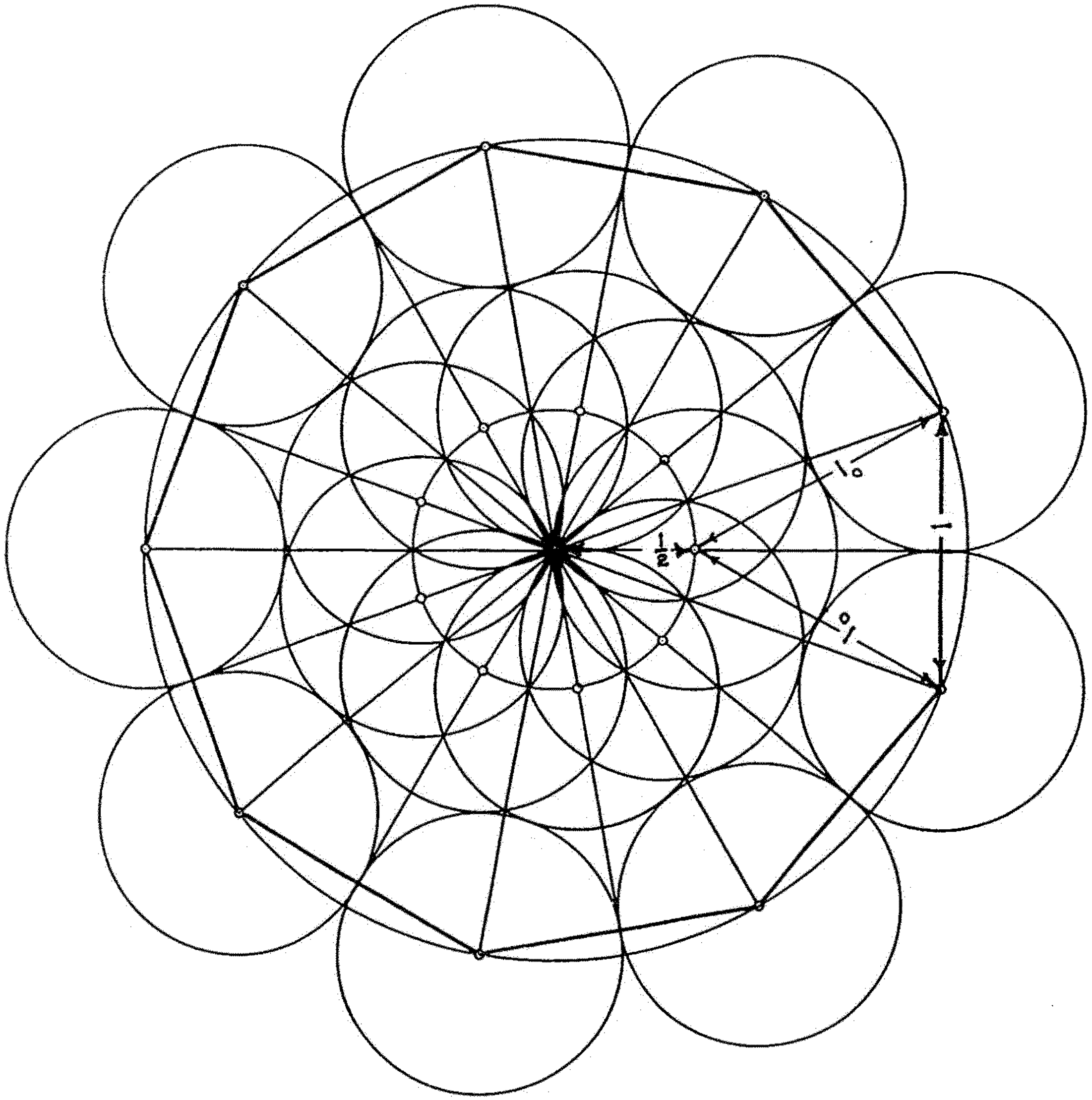
② $\frac{c^2}{4}-\frac{c}{2}+\frac{1}{4}+\frac{c^2}{3}-\frac{1}{4}-\frac{c}{\sqrt{3}}\sqrt{\frac{c^2}{3}-\frac{1}{4}}+\frac{c^2}{12}=1.$

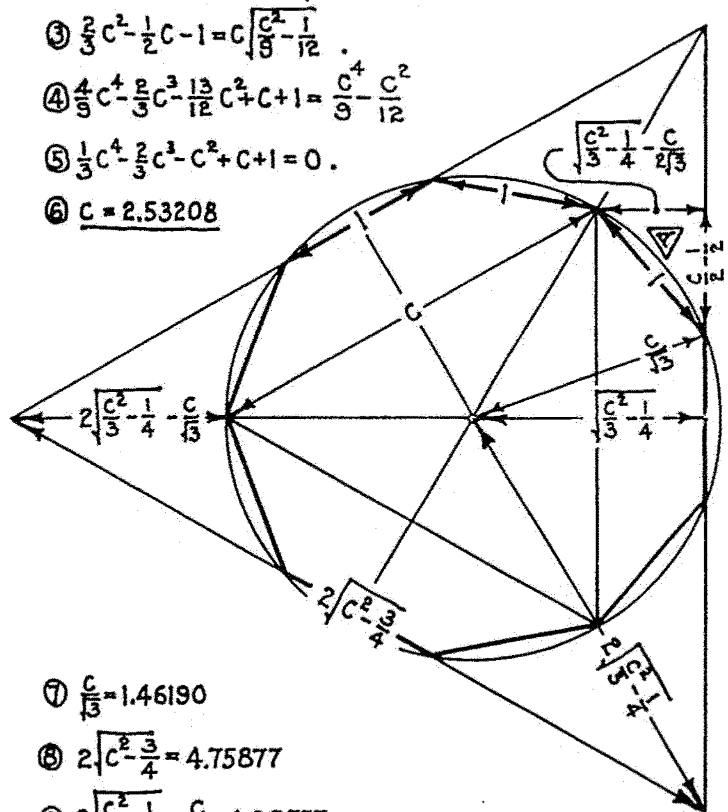
③ $\frac{2}{3}c^2-\frac{1}{2}c-1=c\sqrt{\frac{c^2}{9}-\frac{1}{12}}.$

④ $\frac{4}{9}c^4-\frac{2}{3}c^3-\frac{13}{12}c^2+c+1=\frac{c^4}{9}-\frac{c^2}{12}$

⑤ $\frac{1}{3}c^4-\frac{2}{3}c^3-c^2+c+1=0.$

⑥ $\underline{c=2.53208}$





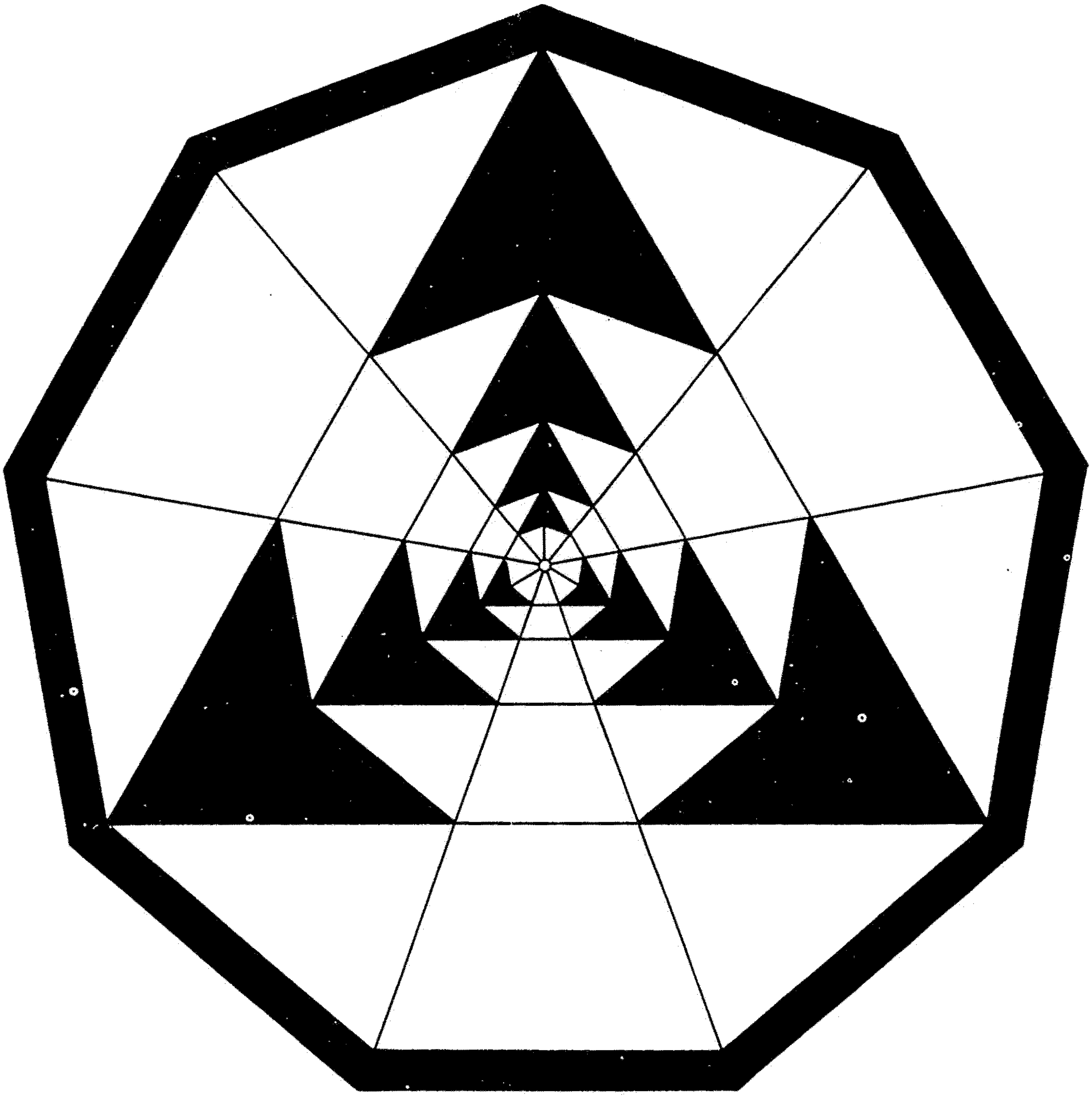
⑦ $\frac{c}{\sqrt{3}}=1.46190$

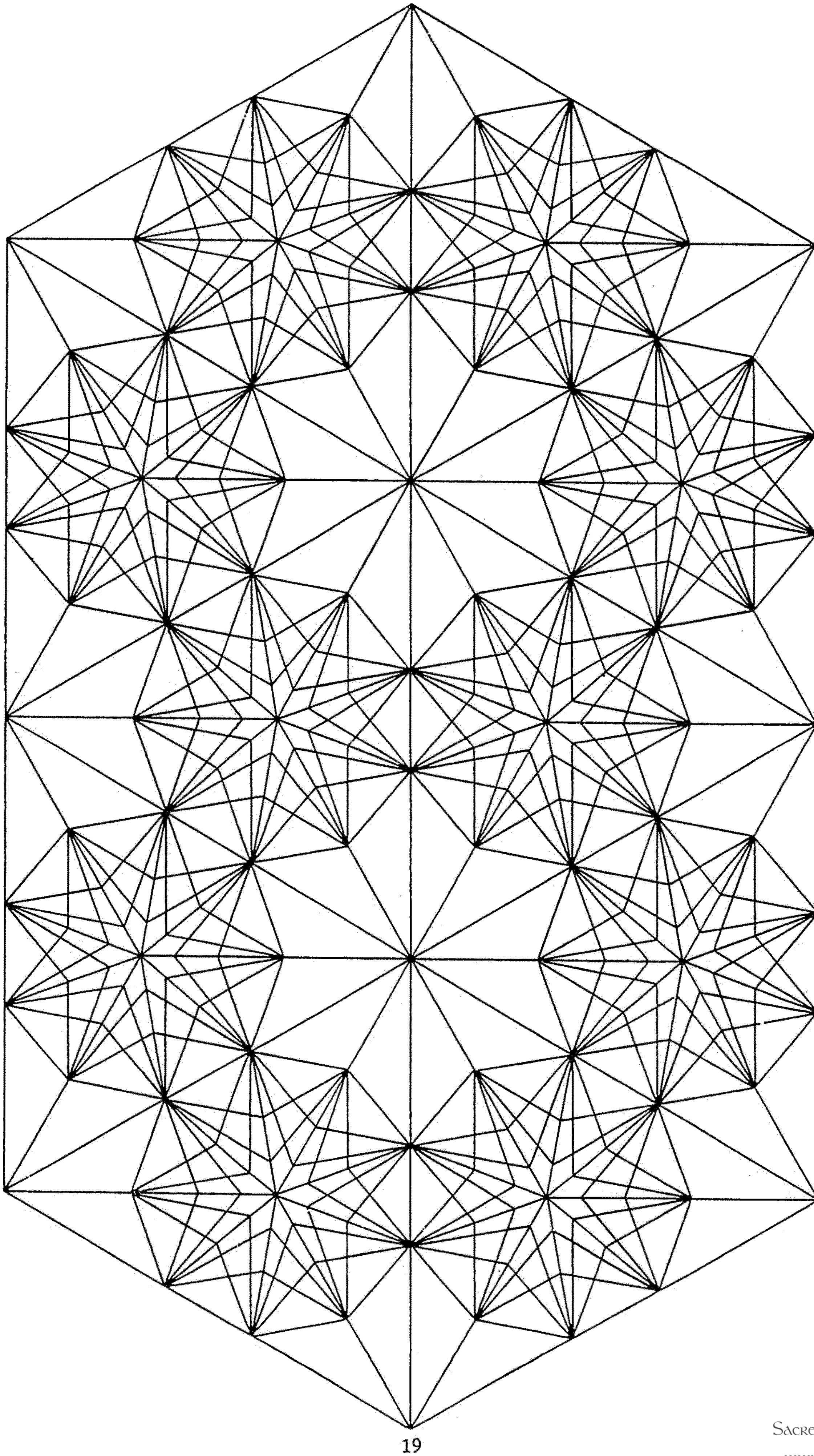
⑧ $2\sqrt{c^2-\frac{3}{4}}=4.75877$

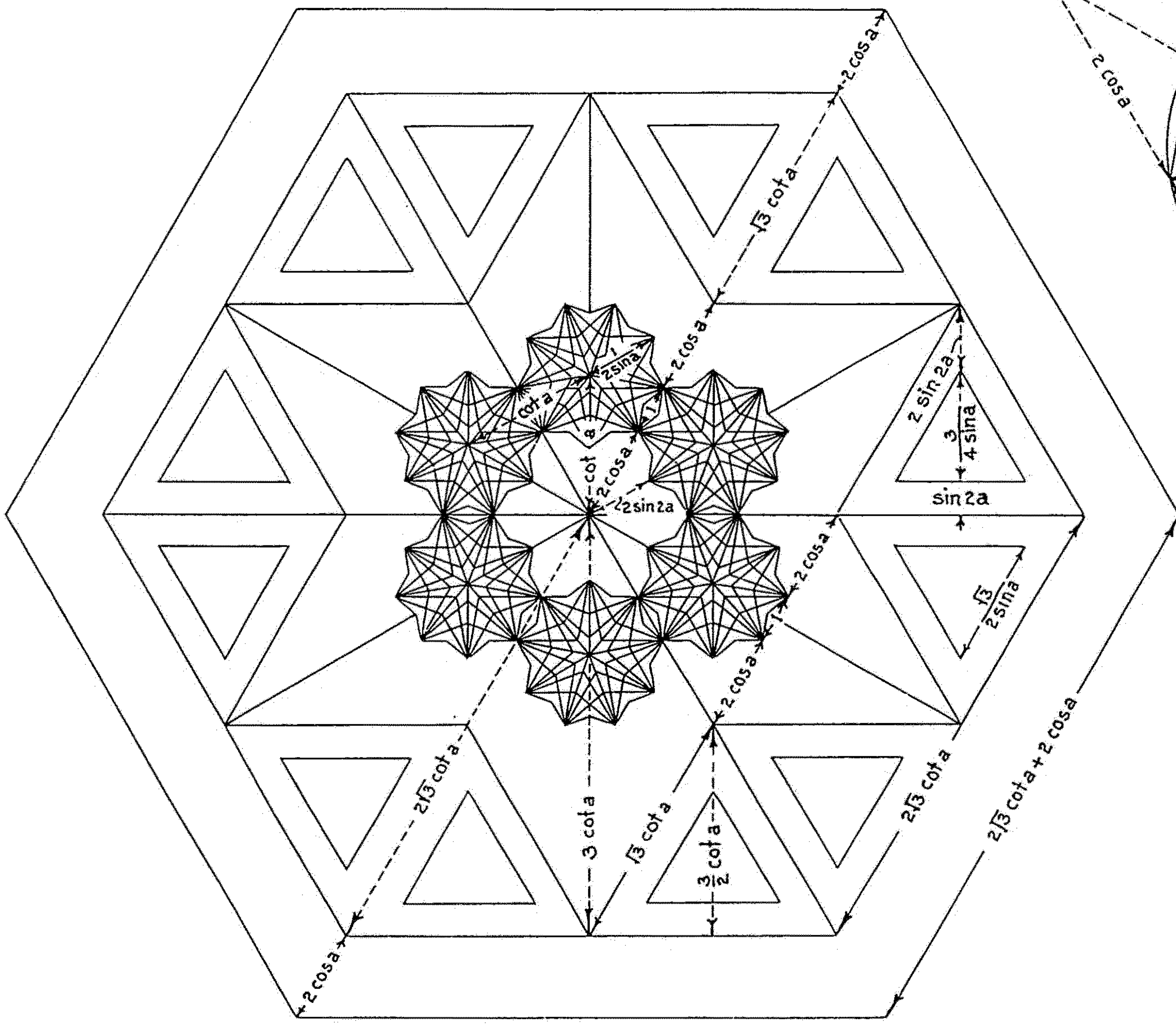
⑨ $2\sqrt{\frac{c^2}{3}-\frac{1}{4}}-\frac{c}{\sqrt{3}}=1.28557$

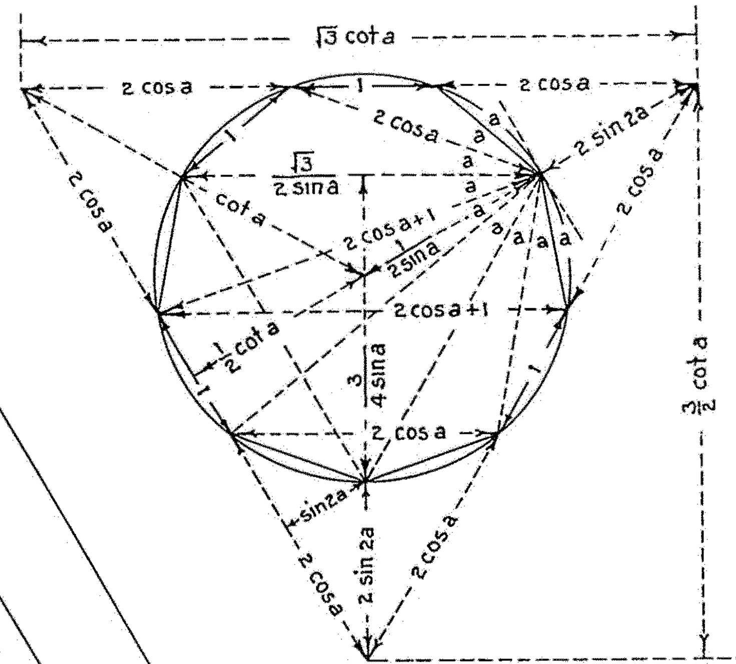
⑩ $\sqrt{\frac{c^2}{3}-\frac{1}{4}}-\frac{c}{2\sqrt{3}}=0.642785.$ $\quad 1_o=1.00669.$

$\frac{3}{c^2}=3-c.$ $\quad (c+1)(c^3-3c^2+3)=0.$ $\quad (c+1)(c-2)^2=1.$ $\quad \frac{c^3}{3}=c^2-1.$ $\quad \sqrt{c+1}=\frac{1}{c-2}.$









$a = \frac{180^\circ}{9} = 20^\circ$		$2a = 40^\circ$.	
$\frac{1}{2 \sin a}$	1.461903	$\frac{1}{2} \cot a$	1.37373
$\frac{3}{4 \sin a}$	2.19285	$\cot a$	2.74747
		$\frac{3}{2} \cot a$	4.12120
$\frac{\sqrt{3}}{2 \sin a}$	2.53208	$\sqrt{3} \cot a$	4.75877
		$3 \cot a$	8.24240
$2 \cos a$	1.87939	$2\sqrt{3} \cot a$	9.51754
$2 \cos a + 1$	2.87939	$\sin 2a$	0.642785
$4 \cos a + 1$	4.75878	$2 \sin 2a$	1.28557

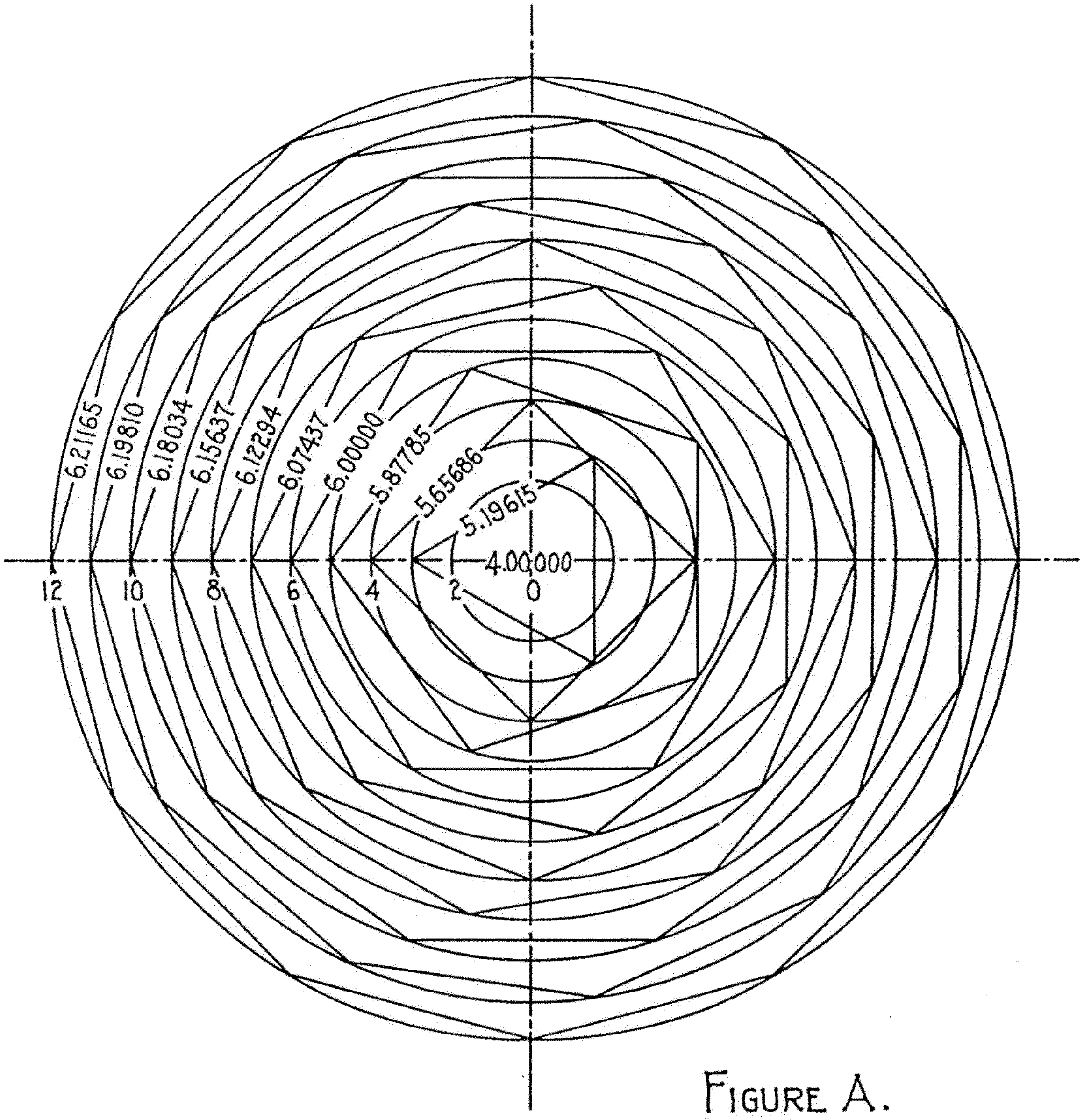


FIGURE A.

THE VALUE OF 2π.

In Figure A, the radius of any circle, n, is n units long and an n sided regular polygon is inscribed in the circle. S_n, the side of the polygon, is measured in terms of the unit subdivisions of the radius. S_n approaches 2π in value as n increases. 2π = 6.28318531.

SIDES OF REGULAR POLYGONS.

$$S_n = 2n \sin \frac{180^\circ}{n}.$$

n.	S_n.	n.	S_n.
2	4.00000	10	6.18034
3	5.19615	11	6.19810
4	5.65686	12	6.21165
5	5.87785	36	6.27520
6	6.00000	360	6.28310
7	6.07437	3600	6.28317
8	6.12294	10800	6.28319
9	6.15637	∞	2π

All S_n arcs are 2π long.