THE PARTHENON
AND OTHER GREEK TEMPLES THEIR DYNAMIC SYMMETRY

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MCMXXIV

NEW HAVEN · YALE UNIVERSITY PRESS

LONDON · HUMPHREY MILFORD · OXFORD UNIVERSITY PRESS

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INTRODUCTION

DYNAMIC SYMMETRY is an ancient Greek term, ***δυνάμει σύμμετρος***, meaning ***commensurable in power***, particularly the second power or square of a number. The term was used to describe the lines which constitute the sides of rectangles bearing a square-root relationship with their ends. These side and end lines are incommensurable (asymmetrical), but the squares on them are commensurable (symmetrical), as 1:2:3:4:5, etc.

The term seems to have been an invention of the early Greeks as it appears in the most ancient fragment of their geometry which has survived. For many hundred years it was used to distinguish the lines mentioned. (See Note I.)

See Euclid X, Def. 2.

Diophantus of Alexandria (fl. A.D. 250) was the first to make use of algebraic symbols. For the powers of the unknown he used $\Delta\gamma$ for **δύναμις**, the square, and $\kappa\gamma$ for ***κύβος***, the cube, and so on.

The analyses of the proportions of the early Greek temples are based upon one of the rectangles of the classification mentioned—that with end of unity and sides equal to the square root of five. It should be remembered that this root-five line is merely a diagonal to two squares.

By a logical process this rectangle is subdivided to produce other rectangles and these, with the generating area, explain the proportions of the plans of the temples in two and three dimensions.

The numerical value of the square root of five is 2.236+. The fraction is endless.

The rectangles obtained by subdividing a root-five area are:

.236, which represents the difference between two squares (2) and 2.236. This fraction is the reciprocal of 4.236. (.236 equals root five minus 2; 4.236 equals root five plus 2; .618 equals root five minus 1, divided by 2; 1.618 equals root five plus 1, divided by 2.)

.618. This fraction is the reciprocal of 1.618.

The last represents the celebrated extreme and mean ratio of the Greeks which was so well known by the time of Plato that it was called "The Section."* In more recent times it was called "The Divine Section" and still later "The Golden Section."

* "We are told by Proclus that Eudoxus 'greatly added to the number of the theorems which Plato originated regarding *the section*, and employed in them the method of analysis.' It is obvious that *the section* was some particular section which by the time of Plato had assumed great importance, and the one section of which this can safely be

The temples which have been analysed in the light of dynamic symmetry are:

The Parthenon, which forms the bulk of this book.

The temple of Apollo at Bassæ in Arcadia by Iktinos, the architect of the Parthenon.

The Zeus temple at Olympia.

The temple at Ægina.

The temple at Sunium near Athens.

Considerable work has been done on the early Greek temples in Sicily and elsewhere, but this material is not ready for publication.

In a previous volume, *Dynamic Symmetry: The Greek Vase,* and in the magazine *The Diagonal*, the small monthly journal containing disconnected material which came to light while the investigation was in progress (published in twelve numbers during 1920-1921), the difference between dynamic and static symmetry was sketched. Therefore it will not be necessary to repeat the distinction. Reference, however, may be made to the architectural plans by Michael Angelo now in the Buonarotti museum at Florence, Italy, and the sketch-books of Leonardo da Vinci and Villars de Honnecort for illustrations of the static type of symmetry used during the middle ages and renaissance.

The static symmetry used by the Romans is very well exemplified by the architectural principles enunciated by Vitruvius. This author's modulus scheme would result automatically in this lower grade of symmetry. He does recommend a root-two proportion for an atrium but apparently did not understand anything of its properties—to him it was probably an echo of a tradition. The fact that the Roman writer supplies us with elaborate explanations of the proportions of Greek buildings and that none of them are verified by the actual remains is rather conclusive proof, it seems to me, that he did not know what they were.

Vitruvius lived in the time of Augustus and the Greek authorities he mentions flourished during the decaying period of classic art when the principles observable in the early architecture had become lost.

During Nero's time, shortly after Vitruvius, the theatre of Dionysos at Athens was rebuilt and arranged for certain barbaric spectacles. The

said is that which was called the 'golden section,' namely, the division of a line in extreme and mean ratio which appears in Euclid 2 : II, and is therefore most probably Pythagorean." *The Thirteen Books of Euclid's Elements*, T. L. Heath, Vol. I, p. 137.

new floor of the orchestra was then decorated with a large pattern made by pavement blocks. This floor is in a fairly good state of preservation, as the photograph shows. (Note II and figure 96.) Its proportions are easily ascertainable and they probably shed as much light upon Roman design thought as anything which has survived. This pattern is just the sort of thing we should expect from the Vitruvian principles, but it is far away from the design of ancient Greece.

The outstanding proportions of this pattern are root three to one (Note II), but, like the root-two proportions of the Roman atrium, there is no indication that its designers knew anything of the properties of the rectangle. I am inclined to believe that the designers of this period were trying to use an old Greek method, the meaning of which had become obscure. The early Greeks used the root-three proportion correctly to a limited extent in certain types of design but not in architecture, if the Choragic Monument of Lysicrates is excepted. To a certain extent therefore we have positive evidence of the use of root rectangles in Roman design much in the same manner that these figures were employed by the artists of the middle ages.

There are a few surviving grave stelæ which throw some light upon the question of the date for the appearance of dynamic symmetry in Greece. One is now in the Museum at Athens, one in the Metropolitan Museum of Art, New York, and one in the Museum of Fine Arts, Boston. The two former seem to have been made at Athens about 550 B.C. The one at Boston is from the Troad and its date is probably about 500 B.C. These three stelæ are examples of dynamic symmetry design.

The example in the Museum at Athens discloses such a simple scheme of dynamic symmetry that it furnishes an admirable illustration of this type of proportion. The measurements, made by well-known authorities, and the calculated proportions are shown in the table.

	Measurements.	*Calculation.*
Width of abacus	0.678 meters.	0.678 meters.
Width of neck	0.420 meters.	0.41903 meters.
Height of abacus	0.129 meters.	0.12949 meters.
Depth of abacus	0.255 meters.	0.2554 meters.
Depth of neck	0.170 meters.	0.1695 meters.

The difference between the symmetry scheme and actual measurement is less than a millimeter for all the details.

The width of the abacus is used as a side of a square to reduce the two dimensional plan to terms of area as is done with all the Greek temples. The symmetry scheme is begun by placing a .618 rectangle in the center of this square. This first step, however, finishes the process as all the proportions of the design are at once established.

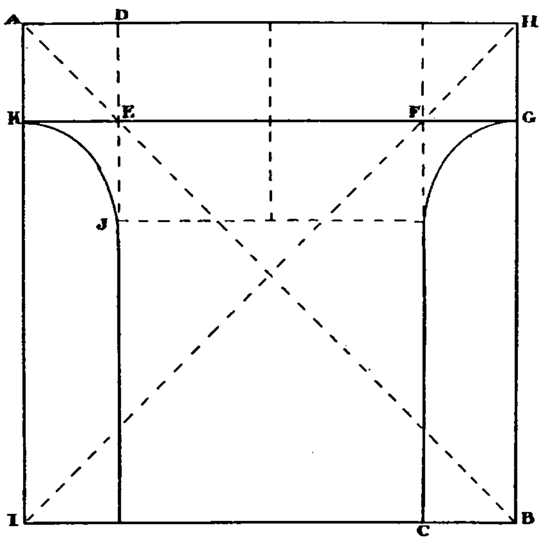

Figure 1.

Diagram of the Grave Stele at Athens.

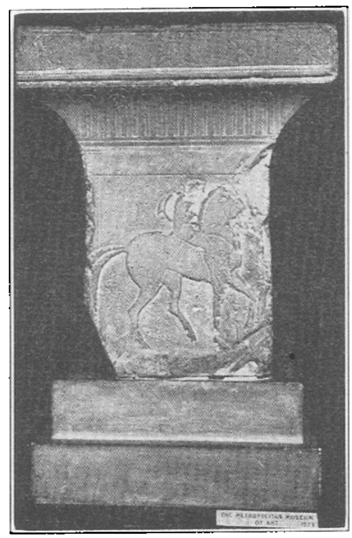

Figure 2.

Grave Stele at Athens.

AB is the square fixed by the abacus width.

CD is the .618 rectangle.

AE, HF are squares.

Through E and F the line KG fixes the abacus AG.

CE is composed of the square CJ and the two .618 areas JF.

DF is composed of two .618 rectangles.

AG, DI, and CH are similar and equal figures, each being composed of a root-five rectangle and three squares.

IE and BF are each composed of a root-five rectangle and two squares.

(The line IB is very close to a moulding on the original, therefore it may be that the plan was intended to be a square.)

It will be noticed that the voids as well as the solids have a proportion value. This is as it should be in a graphic proportioning scheme which fixes symmetry by area and volume rather than by line. The symmetry of the Greek temples is fixed precisely as in this diagram and the proportions of this stele are the actual proportions found in the temples. The entire scheme of dynamic symmetry may be learned from this single example. We might extend the inspection of the stele to the side elevation and thus obtain the three dimensional relations, or fix the plan of a section say at I B (actually a similar figure to the great column-spacing rectangle of the façades of the Parthenon), or the plan of the top at AH, but this is

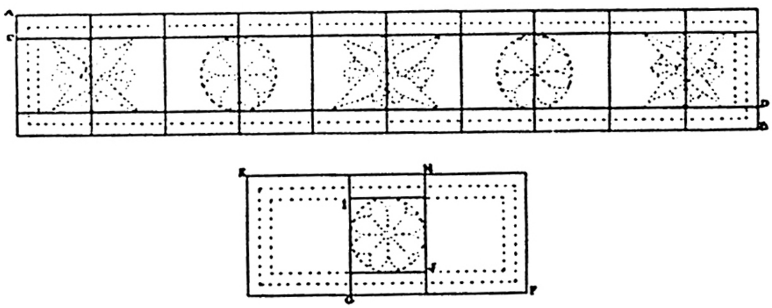

Figure 3.

Decoration on the Abacus of the Stele at Athens.

not necessary for the present purpose as more pertinent material is available in abundance. (See Note III for pottery proportions.)

The abacus of the stele is decorated with a pattern. The arrangement of the units of this pattern is such that they echo the proportions of the design as a whole. The area AB of the abacus face is composed of ten .618 rectangles, while that of CD, the height of which is fixed by the decorative units, is composed of ten squares. Thus each .618 rectangle has a square in its center. The error of this arrangement is extremely small.*

The area of the end is shown by EF, of which HG is a .618 rectangle. The areas EG and HF, together, represent a similar figure to a façade

* An attempt has been made to analyse this stele in terms of static symmetry (see *American Journal of Archæology*, July-September, 1922, pp. 261-277), but as the errors are so considerable—nearly a centimeter at important points in so small an example—the evidence points against rather than for its use.

projection of the Parthenon. It is interesting to note that the area of the façade of the abacus, ten .618 rectangles, appears in the ground plan scheme of the Erechtheum.

The use of two measuring systems, the English and the metric, in the temple analyses may cause some confusion, but I believe it is unavoidable as the most reliable measurements of the Parthenon are those made by the English architect, Francis Cranmer Penrose. He, however, appreciated the weakness of the English system for the purpose and had the foresight to employ a decimal division of the foot. I have had occasion to check Penrose's work and, except in some minor particulars, have found it very accurate.

A peculiarity of the plans of Greek temples of the classic period is the unequal spacing of the front and flank columns. This applies especially to the temples of Greece proper and not so much to those of the colonies. This inequality of spacing, I am inclined to believe, is a sure indication of the use of dynamic symmetry by the ancient architects. If static rectangles were used for planning, the most logical process of subdivision would be in even multiples for fronts and flanks. If no rectangles were employed and the plan and its subdivisions were fixed by length units, whether by a modulus or a foot or cubit, a static result would follow automatically. Using the foot or the meter the modern architect almost invariably makes equal spacing around a building, and so did the architects of the Hellenistic period.

Another peculiarity, of similar character to the irregular column spacing in the classic Greek building, is that of inequality of step heights and treads. Again the modern architect makes these equal, as did the Hellenistic Greek designer.

This is logical if static design is employed, but more or less illogical if a dynamic method of setting out the plan is used. The ends and sides of dynamic rectangles can hardly be divided in equal multiples. If they could be so divided then the entire interior of the plan would be arranged in squares by lines connecting the column centers.

Thus it is clear that whatever explanation is made of classic Greek methods of fixing temple plans the fact of front and flank differences in column spacing and inequalities of step heights and treads must be considered. Up to the present I believe dynamic symmetry offers the only rational explanation of these peculiarities.

CHAPTER ONE: "THE GREEK TEMPLE"

HOW THE BUILDING IS ANALYSED

THERE are two methods of analysis which may be used for a study of the character of the proportions of the Parthenon at Athens.

One employs certain subdivisions of the temple plan which were fixed by the architects, such as areas defined by column centers.

The other consists of the fixing of the proportional character of voids and solids as made by the two or three dimensional plans and elevations.

The second method is sweeping in character and discloses every possible phase of proportional or non-proportional relationship.

The first method is necessarily limited to certain parts of the two dimensional plan; the second embraces every aspect of two and three dimensions.

As was explained in the Introduction there are three rectangles used in the analysis of the temple.

(1) A root-five rectangle with the value of 2.236, reciprocal .4472.

(2) A 1.618 rectangle, reciprocal .618.

(3) A .236 rectangle, which represents the difference between two squares and a root-five area (2. and 2.236). The fraction .236 is the reciprocal of 4.236, that is, root five plus two squares.

When lines are drawn across the floor of the temple from center to center of the regularly spaced columns of the façades, as they are fixed by the centers of the angle columns, the area thus obtained is composed of .618 rectangles arranged in a peculiar manner. The angle columns are larger in diameter than the intermediate columns; hence, the centers of the former are slightly farther away from the edge of the top step than the latter.

There are six regularly spaced columns on each front, consequently the area made by the six strips divides the major part of the temple floor into a trellis of .618 rectangles. (See figure 4.)

A, B, C, D, E, F are centers of the six regularly spaced columns of a façade as fixed by the centers of the angle columns.

GH is composed of two .618 rectangles.

HI is a .618 area. There are eleven of these groups in each of the six strips.

When lines are drawn across the temple floor from center to center of the flank columns, fixed in the same manner as are those of the fronts, the result is a series of area strips each of which is composed of .236 rectangles and squares. (See figure 5.)

AD is a strip fixed by the column centers of the flanks. EF, FG are 2.36 rectangles (note the position of the decimal point), each of which is a similar figure to the column center rectangle of the cella.

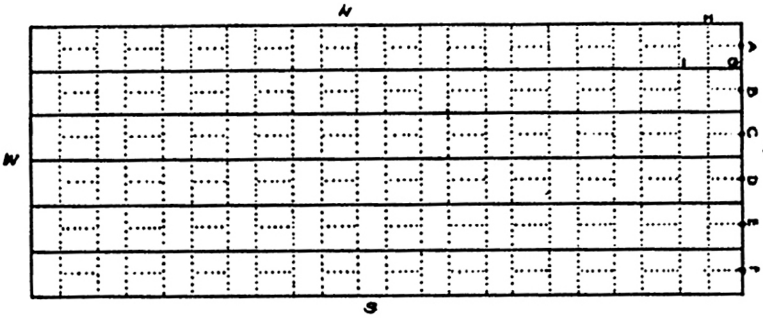

Figure 4.

BE, GD are squares. There are 15 of these strips.

Figures 4 and 5 solve a difficult problem, namely, that of the subtle difference between the widths of the front and flank intercolumniations. (See Appendix, Note IV.)

The centers of the regularly spaced columns of the fronts have a mean separation of 14.1+ feet. (See Appendix, Note V.)

The mean for the flanks is 14.086.

The difference between these two spacings might be thought negligible by the casual reader but the amount is considerable in a length of a couple of hundred feet.

The areas of these two strips are essentially dynamic in their proportions and are part of the design scheme of the building.

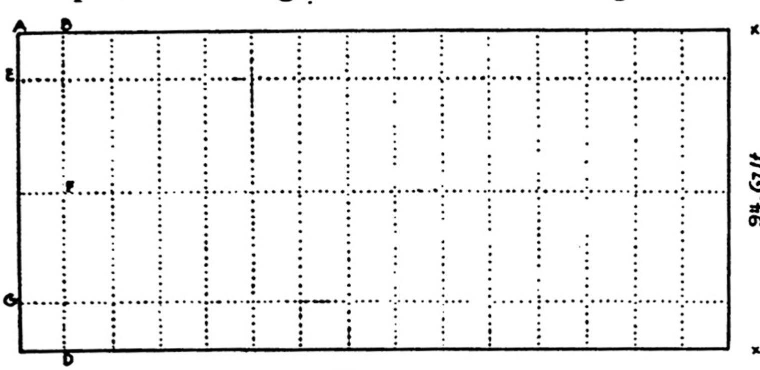

Figure 5.

For the second method of analysis it is necessary to reduce the outstanding proportions of the structure to rectangles and to regard these as subdivided into other rectangles by architectural members such as columns, entablature, pediment or steps by their heights and widths on fronts or flanks. These again may be subdivided into smaller areas until each separate architectural unit is bounded by its own particular rectangle.

The full height and width of the building, for example, furnish an end and side of a rectangle.

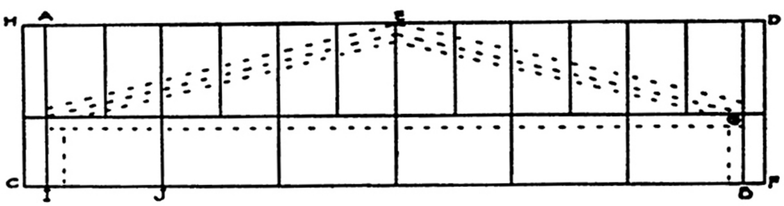

Figure 6.

The full width of a façade is provided by the full front length of the lowest step. Other members of the façade, however, such as entablature and pediment, are shorter than the lowest step; hence, there will be voids as well as solids represented within the rectangle, but the parts of the overall rectangle occupied by these voids must have the same proportional character as the rest. The analysis will, therefore, be an exhaustion of the major rectangle by the void and solid projections. The process may be carried to any degree of refinement necessary.

Figure 6 is an illustration of the second method.

The full length of the entablature, with its corona and mouldings, and the height of the entablature and pediment combined, provide a rectangle composed of .618 rectangles arranged like those of figure 4 (AJ is similar to one of the groups of figure 4.)

The cornice is shorter than the lowest step by an amount shown by the width of the rectangles AC and BD. These two small areas are composed of .618 and .236 rectangles.

The major rectangle of the diagram, HF, is composed of two root-five rectangles, CE and EF.

Hence the entablature and pediment projection provide a rectangle, AB, within a double root-five rectangle HF, and the difference between the two is represented by the areas AC and BD.

THE GROUND PLAN

THE ground plan of the Parthenon is fixed by its lowest step. This is the first levelling course, known as the *euthynteria*. The width of the end of this rectangle is 111.31, the length of a side is 238.003± English feet. These measurements are demanded by the trellis scheme which has been found in the plan. They differ from the direct measurements obtained by experts by less than one-quarter of an inch in the length. The width is exact. One side of the building is slightly longer than the other, however, by an amount which totally eliminates the error.

The area of the ground plan rectangle is composed of two squares and two root-five rectangles arranged as in figure 7.

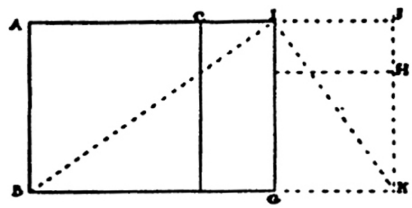

Figure 7.

BC, GH are squares, while CG, IH are root-five rectangles.

The square and root-five area IK is fixed by the right-angled triangle BIK.

THE FAÇADE

PENROSE measured and calculated the height of the Parthenon from the ground to the top of the sima. He was not able to measure all the members as the roof and the upper parts of both pediments were destroyed.

The following is a comparison of the actual measurements with those demanded by the pattern trellis. (Measurements in feet and thousandths.)

	Penrose.	*Dynamic Trellis.*
Steps	6.058	6.025
Column (angle)	34.253	34.247
Entablature	10.793	10.779
Pediment	14.079 (calculated)	14.111
Total	65.183	65.162

The greatest difference is that of the pediment height, which Penrose was unable to measure but assumed that it was equal to one intercolumniation. This height does suggest a façade intercolumniation, and the dynamic figures are much closer than those of Penrose, but I leave the question open.

The column heights are for those at the angles. The middle ones are shorter by something less than half an inch. This difference is due to the fact that the horizontal curvature of the epistyle is less than that of the stylobate upon which the columns stand.

The most striking aspect of the façade rectangle, fixed at 65.162 by 111.31 feet, is that it is composed of a square and three .236 rectangles.

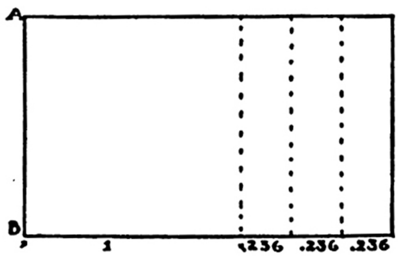

Figure 8.

Another interesting feature is that the bottom of the epistyle, or top of the columns, divides the area of the façade projection in extreme and mean ratio. AB is thus divided at C (figure 9).

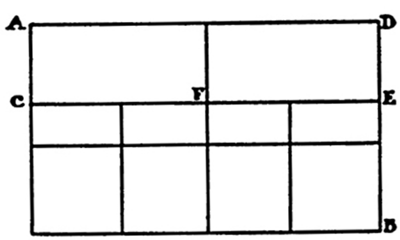

Figure 9.

The bottom of the epistyle, the line CE of the diagram, divides the façade rectangle into squares and root-five areas.

AE is composed of two root-five rectangles, AF, FD.

CB is composed of four squares and four root-five rectangles.

The diagram is a comparison of the rectangles of the façade and ground plan by imposing one rectangle on the other.

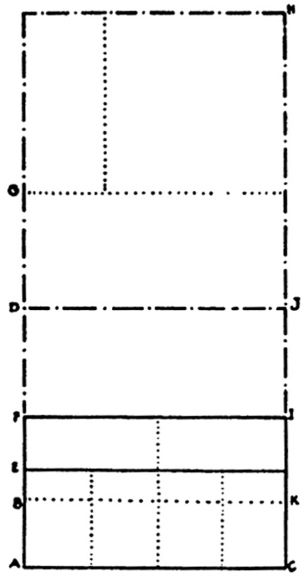

Figure 10.

AH is the ground plan area.
AJ is the ground plan square.
AI is the façade rectangle.
AK are the four squares of the façade.
AB is one-fourth of AD.
AE is one-fourth of AG.
EF is one-half of DG.
BE is one-half of EF.
GH is the reciprocal of CG.

The bottom line of the epistyle, probably the most noticeable division of the building, brings into prominence the celebrated Greek proportion referred to by the ancient authorities as "The Section." This line divides the elevation of the structure into two parts on fronts and flanks which, individually and collectively, bear to each other this extreme and mean relationship. The area AE of figure 9 is to the area CB as is .618 to 1.

The area CB of figure 11 bears the same relation to AD, etc.

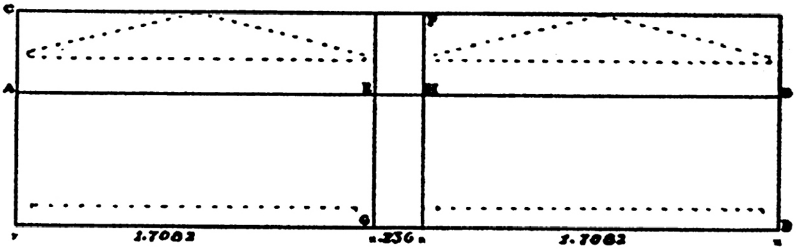

Figure 11.

The flank elevation of the Parthenon. Equal to two façades plus a .236 figure.

THE FLANK ELEVATION

THE flank elevation, 65.162 by 238.003 feet, furnishes an area composed of two rectangles of the façade plus a .236 figure (figure 11).

CG and FD are each equal to a façade rectangle.

GF is a .236 rectangle.

CE and FB are each composed of two root-five rectangles.

FE is a .618 rectangle.

AG and HD are each composed of four squares and four root-five areas.

GH is composed of a 1.618 area plus a square.

As the method of analysis is to begin with the outstanding proportions, consideration of details follows in succeeding chapters.

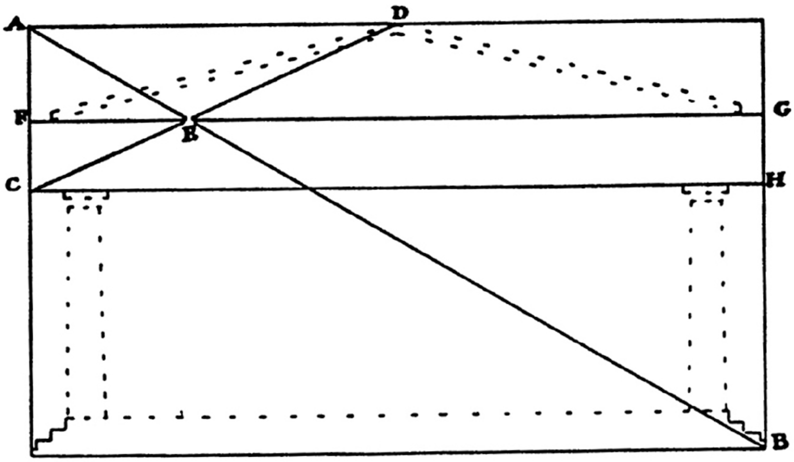

Figure 12.

CHAPTER TWO: THE FAÇADE AND ITS DIVISIONS

AB (figure 12) is the rectangle of the façade.

CH is the top of the columns.

CD, DH are root-five rectangles.

(See figure 9 for façade construction.)

AB is a diagonal to the entire rectangle; CD a diagonal to a root-five area.

These lines intersect at E.

FG, drawn through E, is the floor of the pediment or the top of the entablature cornice.

AG is the rectangle for the entire pediment projection while FH is that for the entablature.

The area AG is readily analysed by the diagonal lines AE, ED.

FD is half of the area AG.

AE is a similar figure to AB while ED is a root-five rectangle.

Thus AG is composed of two similar figures to a façade rectangle plus two root-five rectangles.

FE of figure 13 represents the area of the complete entablature projection.

It is composed of four root-five rectangles plus the area HI.

This latter area is composed of two squares and two root-five rectangles.

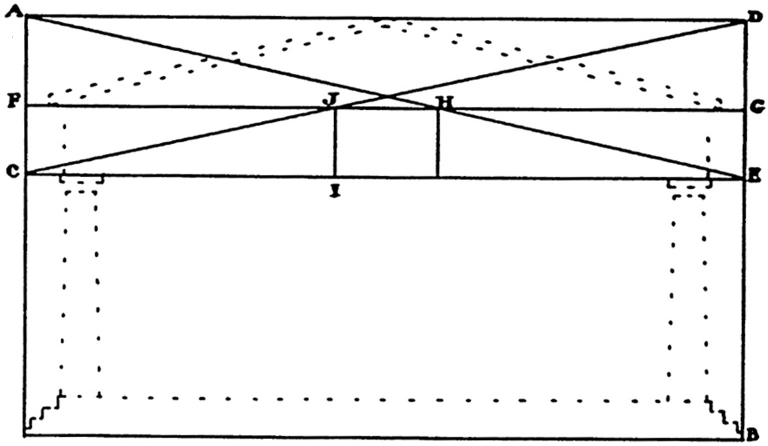

Figure 13.

The façade rectangle, without that of the pediment, AB of figure 14, is composed of two .618 rectangles plus four .236 areas.

AC equals .618 x 2.

CD equals .236 x 4.

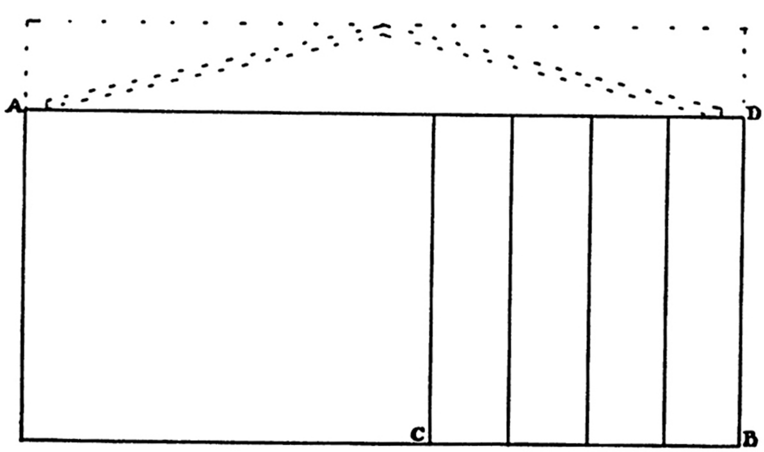

Figure 14.

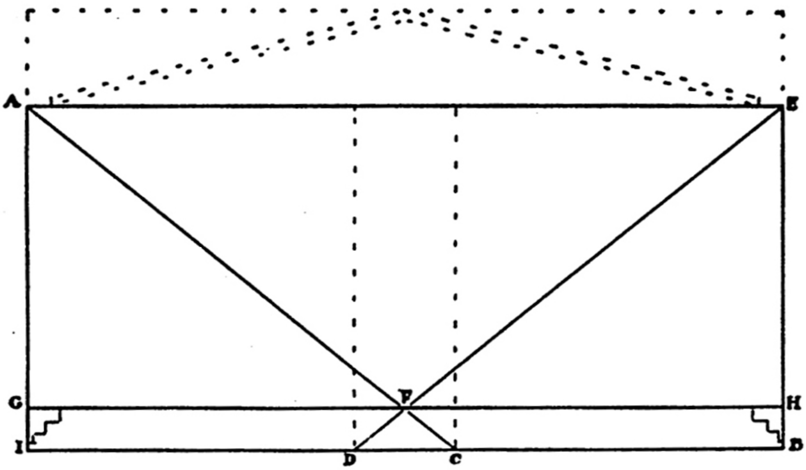

Figure 15.

AE of figure 15 is the top of the entablature while I B is the ground.

AC is composed of two .618 rectangles while DE is a similar and equal figure which overlaps AC to the extent of CD.

Diagonals to these overlapping areas cross each other at F.

GH, through F, is the top line of the steps.

The area AH is composed of four .618 rectangles.

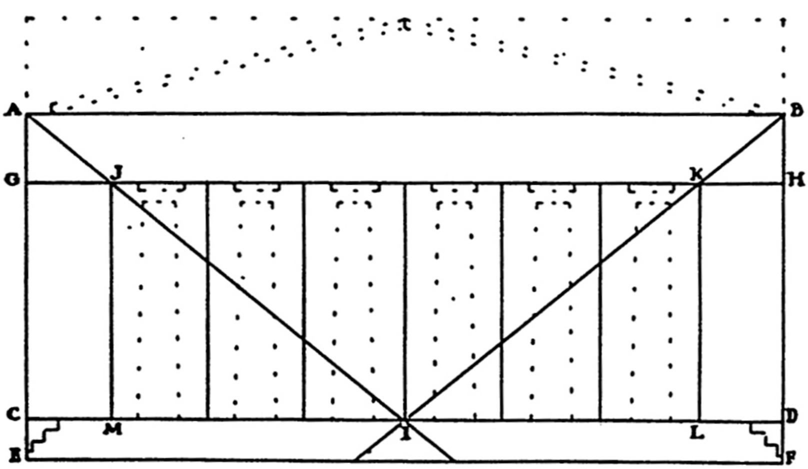

Figure 16.

AB of figure 16 is the top of the entablature.

CD is the top of the steps.

GH is the top of the columns.

AD is composed of four .618 rectangles fixed by the diagonal lines AI, IB.

These lines cut the line at the top of the columns, GH, at J and K.

The rectangle MK is composed of four .618 rectangles. It is also the rectangle which exactly contains the six regular intercolumniations of

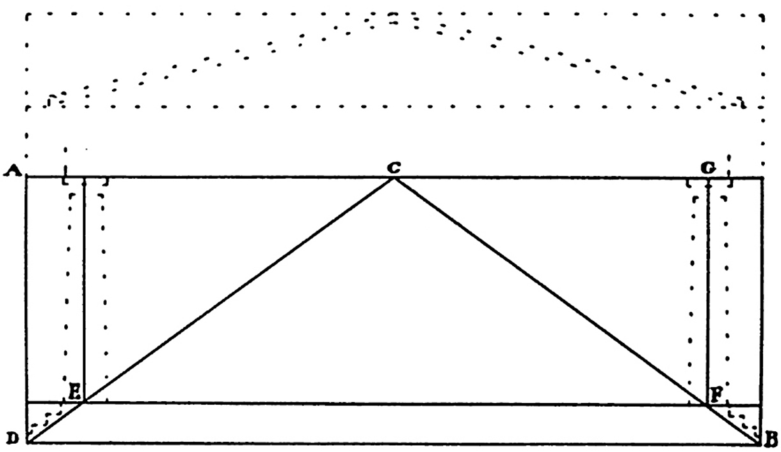

Figure 17.

each façade when a column is placed in the center of each of the six rectangles into which MK is divided.

This rectangle provides the centering of all façade columns except those of the angles. (See figure 16.)

AB of figure 17 is the façade rectangle without the entablature and pediment.

It is composed of four squares and four root-five rectangles. (See figure 9, Chapter One.)

CD and CB are each diagonals to one-half of AB.

These diagonal lines cut the top step line at E and F.

The points E and F are the centers of the angle columns.

Thus the rectangle containing all the column centers of each façade is composed of four squares and four root-five rectangles, because EG is a similar figure to AB.

Six regular column spacings fill the rectangle MK of figure 16.

If this rectangle is divided vertically into six equal parts each will be a

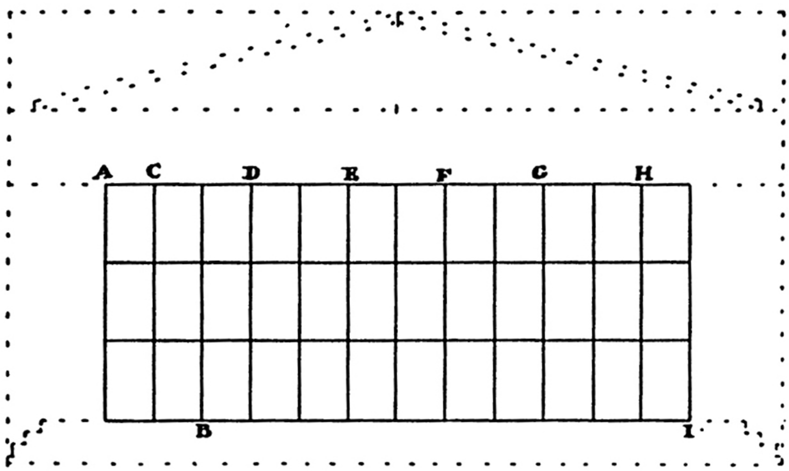

Figure 18.

rectangle composed of two squares and an area similar to one-quarter of the façade rectangle, or;

The entire area AI of figure 18 may be considered as a trellis of .618 rectangles when it is divided horizontally into three equal parts and vertically into twelve equal parts.

A column occupies a space in the center of each one-sixth of AI, as AB. C, D, E, F, G, and H are column centers.

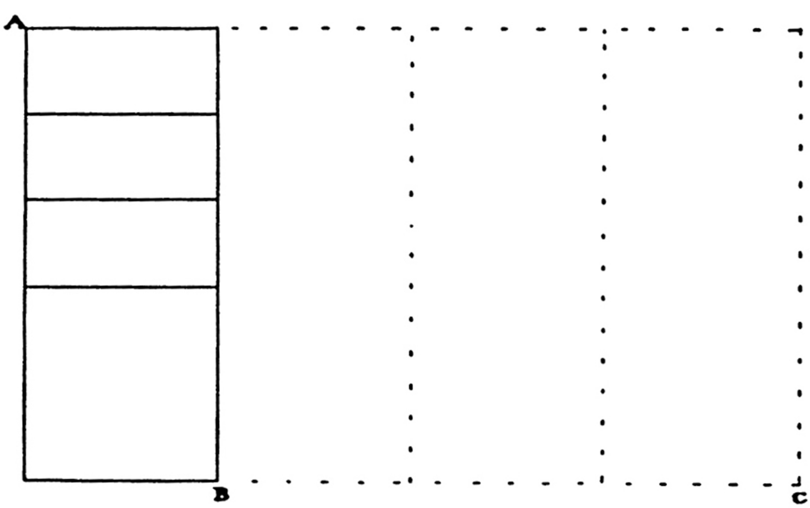

Figure 19.

One-quarter of the façade rectangle (figure 19), as AB of AC, is composed of a square and three root-five rectangles.

The height of the steps with the full width of the building provides a rectangle composed of eighteen squares and two .236 areas.

The rectangle made by the column heights with the full width of the building is composed of three squares and a quarter.

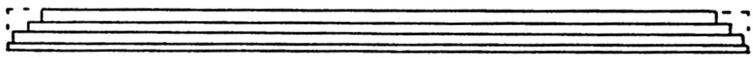

Figure 20.